TECHNICAL RELEASE

NUMBER 67

REINFORCED CONCRETE STRENGTH DESIGN

AUGUST 1980

U. S. DEPARTMENT OF AGRICULTURE

SOIL CONSERVATION SERVICE

NATIONAL ENGINEERING STAFF

DESIGN UNIT

Preface

For the past twenty some years, the structural engineering profession has been gradually adopting the strength design approach in lieu of working stress design. This technical release presents criteria and procedures for design of reinforced concrete structures and structural elements by the strength design method. Working stress design is continued as an acceptable alternate for an interim period.

Design philosophy holds that at every section, design strength must equal or exceed required strength. Design provisions attempt to ensure ductile behavior at ultimate load and adequate serviceability at working load. The American Concrete Institute Standard ACI (318-77) is taken as the basic reference. The ACI is oriented toward the design of buildings. Adjustments and modifications are made herein to reflect the more difficult environment encountered by Service hydraulic structures. The intent of these adjustments and modifications is to provide criteria that will yield design sections that are in essential agreement with sections obtained from current Service working stress design. With time, experience with strength design will indicate further changes are warranted.

A draft of the text portion of the subject technical release dated August, 1979 was circulated through the Engineering Division and was sent to the Technical Service Center Design Engineers for their review and comment.

This technical release was prepared by Edwin S. Alling, Head, Design Unit, National Engineering Staff, Glenn Dale, Maryland. Mrs. Joan Robison contributed much to the layout and preparation of the ES-drawings. Mrs. Dorothy A. Stewart typed the technical release.

TECHNICAL RELEASE
NUMBER 67

REINFORCED CONCRETE STRENGTH DESIGN

Contents

Engineering Standard Drawings

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN:

NOMENCLATURE

A ≡ in crack control, the effective tension area of concrete per bar

A_b ≡ area of individual reinforcing bar

A_g ≡ gross concrete area of section

A_ℓ ≡ area of longitudinal reinforcement resisting torsion

A_s ≡ area of tension reinforcement

A'_s ≡ area of compression reinforcement

A_{sf} ≡ area of reinforcement required to develop compressive strength of overhanging flanges

A_{st} ≡ total area of longitudinal reinforcement

A_t ≡ area of one leg of a closed stirrup resisting torsion within a distance s

A_v ≡ area of shear reinforcement in a distance s

a ≡ depth of equivalent rectangular stress block

a_b ≡ depth of equivalent rectangular stress block at balanced strain conditions

b ≡ width of compression face of member

b_w ≡ width of web

C ≡ any combination of loads, except dead, giving a maximum requirement

C_t ≡ a factor relating torsion and shear forces to torsion and shear stresses

c ≡ distance from extreme compression fiber to neutral axis

c_b ≡ distance from extreme compression fiber to neutral axis at balanced strain conditions

D ≡ dead load

d ≡ distance from extreme compression fiber to centroid of tension reinforcement

d' ≡ distance from extreme compression fiber to centroid of compression steel

d'' ≡ distance from mid-depth of section to centroid of tension reinforcement

d''' ≡ distance from plastic centroid of section to centroid of tension reinforcement

d_b ≡ nominal diameter of bar

d_c ≡ thickness of concrete cover measured from the extreme tension fiber to the center of the longitudinal bar located closest to the extreme fiber

d_{min} ≡ minimum depth, d, for which the compression steel can reach yield stress

E_c ≡ modulus of elasticity of concrete

e_b ≡ eccentricity, at balanced strain conditions, of the direct force measured from the plastic centroid of the section

f'_c ≡ compressive strength of concrete

f_h ≡ tensile stress developed by standard hook

f_r ≡ modulus of rupture of concrete

f_s ≡ stress in reinforcement at service loads

f_y ≡ yield strength of reinforcement

h ≡ thickness of two-way slab

J ≡ polar moment of inertia of section

I_{cr} ≡ moment of inertia of cracked section, transformed to concrete

I_e ≡ effective moment of inertia of section for computation of deflections

I_g ≡ moment of inertia of gross concrete section about centroidal axis and neglecting reinforcement

LF ≡ load factor

ℓ ≡ span length of beam or slab

ℓ_a ≡ additional embedment length at support or at point of inflection

ℓ_b ≡ basic development length

ℓ_d ≡ development length

ℓ_e ≡ equivalent embedment length of a hook

ℓ_n ≡ clear span in long direction of slab

M_a ≡ maximum moment in member at loading for which deflection is computed

M_b ≡ nominal moment strength at balanced strain conditions, moments taken about plastic centroid

M_c ≡ service combined load moment

M_{cr} ≡ cracking moment of section

M_d ≡ service dead load moment

M_n ≡ nominal moment strength

M_{ns} ≡ equivalent nominal moment strength about the tensile steel

M_o ≡ nominal moment strength in pure flexure, moments taken about the plastic centroid

M_u ≡ factored moment, or required strength in terms of moment

N_u ≡ required direct force strength acting simultaneously with V_u, taken positive for compression and negative for tension

n ≡ modular ratio of elasticity

P' ≡ smaller of 0.10 $f'_c A_g$ or P_b

P_b ≡ nominal direct force strength at balanced strain conditions

P_n ≡ nominal direct force strength

P_o ≡ nominal compressive direct force strength in pure direct loading, moments taken about the plastic centroid

P_u ≡ required direct force strength

r ≡ in elastic torsional theory for circular cross sections, the distance from the centroid of section

s ≡ spacing of shear or torsion reinforcement in direction parallel to longitudinal reinforcement

T ≡ torque or torsional moment

T_c ≡ nominal torsional moment strength provided by the concrete

T_n ≡ nominal torsional moment strength

T_s ≡ nominal torsional moment strength provided by the torsional reinforcement

T_u ≡ required torsional moment strength

t ≡ thickness of compression flange, slab thickness

U ≡ required strength to resist factored loads

V_c ≡ nominal shear strength provided by the concrete

V_n ≡ nominal shear strength

V_s ≡ nominal shear strength provided by the shear reinforcement

V_u ≡ required shear strength

v_t ≡ torsional shear stress

w ≡ unit weight of concrete

w_c ≡ service uniform combined load

w_d ≡ service uniform dead load

w_u ≡ factored uniform dead load

x ≡ shorter dimension of a rectangular component of the cross section

x_1 ≡ shorter center-to-center distance of the closed rectangular stirrup under consideration

y ≡ longer dimension of a rectangular component of the cross section

y_1 ≡ longer center-to-center distance of the closed rectangular stirrup under consideration

z ≡ eccentricity of the direct force measured from the centroid of the tensile reinforcement

α_t ≡ a torsional coefficient as a function of y_1/x_1

β_b ≡ ratio of area of reinforcement cut off to total area of tension reinforcement at the section

β_1 ≡ ratio of depth of equivalent rectangular stress block to the distance from the extreme compression fiber to the neutral axis

ξ ≡ coefficient for tensile stress developed by standard hook

ρ ≡ tension steel ratio, A_s/bd

ρ' ≡ compression steel ratio, A'_s/bd

ρ_b ≡ tension steel ratio producing balanced strain conditions

$\bar{\rho}_b$ ≡ tension steel ratio producing balanced strain conditions in a rectangular section with tension steel only under flexure without direct force

ρ_{bm} ≡ minimum tension steel ratio for flexural members except slabs

ρ_f ≡ steel ratio of reinforcement required to develop compressive strength of overhanging flanges

ρ_{max} ≡ maximum tension steel ratio permitted under flexure without direct force

ρ_{shy} ≡ maximum tension steel ratio permitted for Service hydraulic structures in a rectangular section with tension steel only under flexure without direct force

ρ_t ≡ temperature and shrinkage steel ratio, A_s/bt

ϕ ≡ strength reduction factor

TECHNICAL RELEASE
NUMBER 67

REINFORCED CONCRETE STRENGTH DESIGN

General

This technical release contains minimum criteria and procedures for use in reinforced concrete design practice in the Soil Conservation Service. The American Concrete Institute Standard, "Building Code Requirements for Reinforced Concrete," ACI(318-77) serves as the basic reference. Its provisions apply as the general design code for the Service except as modified or otherwise stated herein, either directly or indirectly. This technical release supplements, expands on, and/or emphasizes subject matter particularly pertinent to Service requirements. The release is directed toward nonprestressed, cast-in-place construction. It treats neither precast concrete nor prestressed concrete although some aspects of the design of each do apply. References to ACI(318-77), herein referred to as the Code, are by code chapter and section numbers, for example (8.3.1).

The strength design method presented herein, is recommended for use. Alternatively, the working stress design method contained in National Engineering Handbook, Section 6, "Structural Design," subsection 4., "Reinforced Concrete" may be applied using service loads.

The strength design method requires that service loads or related moments and forces be increased by specified load factors and that computed nominal strengths be reduced by specified strength reduction factors. The basic requirement for strength design may thus be expressed as:

Design strength $\geq$ Required strength.

The "required strength" is computed by multiplying the service loads by load factors. Load factors provide for excess load effects from sources such as overloads and simplified structural analysis assumptions.

The "design strength" of an element is computed by multiplying the "nominal (or ideal) strength" by a strength reduction factor, ϕ. The strength reduction factor provides for the possibility that small adverse variations in material strengths, workmanship, and dimensions may combine to result in understrength.

The strength design method not only provides for adequate strength to support the anticipated factored loads, it also includes provisions to assure adequate performance at service load levels. Serviceability provisions include consideration of deflections, crack control, distribution of reinforcement, and development of reinforcement.

The word "service" as used herein has several connotations. When used with upper case, it refers to the Soil Conservation Service. When used with lower case, the reference is to either design life or working loads, forces, moments, or stresses. In some usages, service stresses are in excess of normal Service working stress values.

Structure Environment Class

For purposes of reinforced concrete structural design, the concept of structure environment class, based on the type of environment encountered, is introduced. Two classes are defined herein. These are "Service hydraulic structure" and "other structures." These two classes are not all inclusive. That is, additional classes might be defined for structures subjected to environments that are even more harsh than that of normal hydraulic structures.

A Service hydraulic structure is defined to be any structure subjected to hydrostatic or hydrodynamic pressures, either externally or internally.

Most Service structures are hydraulic structures. As such they are subjected to relatively severe environments. This includes not only hydraulic flows, wave action, and submergence; but also associated combinations of wet/dry and freeze/thaw cycles, and general exposure to the elements. These conditions vary considerably from, and are harsher than, those encountered in the normal building frames envisioned by the Code. Hence not only are structural external stability and internal strength of prime importance, but these hydraulic structures must also have satisfactory durability, adequate crack control, and generally good serviceability characteristics. Because of these requirements, it is essential that design be performed with great care. As noted, the criteria and procedures contained herein are minimums. On occasion it will be desirable to follow more conservative approaches.

Waste treatment works that require a relatively long service life, provide valid examples of structures warranting special considerations to ensure adequate durability in their difficult environments. Other special structures exist that involve unique site, design, and/or construction problems which are not covered by the provisions of this technical release.

Materials

The choice of concrete strength and grade of reinforcing steel to be used in any specific job should be based on a study of the job requirements including strength, durability, service requirements, costs, and the availability of materials, work-force, and equipment. Many factors affect the quality of reinforced concrete. The best materials and design do not produce excellent concrete without high quality methods of construction.

Concrete

The general shapes of stress-strain curves for concrete cylinders and for compression faces of beams are essentially identical. The first part of the curve is nearly straight with appreciable curvature beginning at a unit stress of about half the maximum value. Curvature increases with increasing stress until the peak stress, f'_c, is reached at a compressive strain between 0.0015 and 0.002. At strains beyond the peak value of stress, considerable strength remains. The stress-strain curve descends from the peak stress to an ultimate strain of from 0.003 to about 0.0045. The maximum usable strain for design is set at 0.003.

Nine classes of concrete are currently estabilished. They cover the various conditions of design and construction normally encountered by the Soil Conservation Service. The number associated with the class is, f'_c, the specified compressive strength of the concrete in psi. For Class 5000, Class 4000, Class 3000, and Class 2500 concrete the contractor is responsible for the design of the concrete mix. For Class 5000X, Class 4000X, Class 3000X, Class 3000M, and Class 2500X concrete the engineer is responsible for the design of the concrete mix. The following is a general guide to these concrete classes and their use.

<u>Class 5000 or 5000X concrete</u> -- for special structures, for precast or prestressed construction, for extreme exposure conditions.

<u>Class 4000 or 4000X concrete</u> -- for standard types and sizes of structures, for moderate exposure conditions.

<u>Class 3000 or 3000X concrete</u> -- for small simple structures, for mass foundations.

<u>Class 3000M</u> -- for minor concrete structures in which the quantity of concrete is less than 5 yards and where the location of the concrete will permit easy maintenance or replacement.

<u>Class 2500 or 2500X concrete</u> -- for small structures built by unskilled labor, for plain concrete construction.

Guide Construction Specifications 31. Concrete and 32. Concrete for Minor Structures state the technical and workmanship requirements for the operations required in concrete construction. These specifications include such items as:

Air Content and Consistency
Design of Concrete Mix
Inspection and Testing
Mixing, Conveying, Placing, Consolidating, and Curing Concrete
Preparation and Removal of Forms
Measurement and Payment.

Guide Material Specifications 531. Portland Cement, 522. Aggregate for Portland Cement Concrete, and others state the quality of materials to be incorporated in the construction.

Steel Reinforcement

Reinforcing bars are manufactured from billet steel, rail steel, and axle steel. Stress-strain curves for reinforcement typically consist of four portions. These are: first, an initial, essentially straight-line portion where stress is proportional to strain up to the yield strength, f_y; second, a horizontal portion where stress is independent of strain up to the beginning of strain-hardening; third, another portion where stress again increases with strain, but at a slow rate with the curve flattening out as the tensile strength is reached; and fourth, a last portion where the curve turns down until fracture occurs. The length of the horizontal portion decreases with the higher strength steels. Thus steel ductility tends to decrease with increasing strength.

For design, the stress-strain curves are idealized as consisting of two straight lines. Stress is proportional to strain below the yield strength. Stress is independent of strain at strains exceeding the yield strain. Any increase in strength due to the effect of strain-hardening is neglected in strength computations.

Three grades of deformed reinforcing bars are currently established. These are Grade 40, Grade 50, and Grade 60. The number associated with the grade is, f_y, the specified yield strength of reinforcement in ksi. Deformed bars with a yield strength exceeding 60,000 psi are not permitted. Guide Construction Specification 34, Steel Reinforcement states the technical and workmanship requirements for fabricating and placing reinforcement. Guide Material Specification 539. Steel Reinforcement states the quality of material required. ASTM Specifications A615, A616, and A617 are referenced in Specification 539.

General Requirements and Assumptions

Requirements

Except as already noted, the strength design method shall be used. All structures and structural members shall be proportioned for adequate strength in accordance with the provisions contained herein, using load factors and strength reduction factors as specified. All sections are to have design strengths at least equal to their required strengths. Members shall also meet all other requirements contained herein to ensure adequate performance at service load levels.

All members and/or elements of statically indeterminate structures shall be designed for the maximum effects of factored loads as determined by the theory of elastic analysis; the only exception is the limitation on the maximum torsional moment that need be developed. Factored loads are service loads multiplied by appropriate load factors. Normal simplifying assumptions as to relative stiffnesses, span lengths, arrangements of live loads, etc. may be used.

Assumptions

The strength of a member or element computed by the strength design method requires that two basic conditions be satisfied: (1) static equilibrium and (2) compatibility of strains. Equilibrium between the internal stresses and external forces acting on a free body must be satisfied for the nominal strength condition. Compatibility between stress and strain for the concrete and for the reinforcement must also be established within the design assumptions that follow.

Strain in the reinforcing steel and in the concrete shall be assumed directly proportional to the distance from the neutral axis except that for deep flexural members a nonlinear distribution of strain is appropriate.

Strictly speaking, concrete has no fixed modulus of elasticity. The secant modulus definition of modulus of elasticity is generally used in service load level stress or deflection calculations. The modulus of elasticity of concrete, E_c, may be taken in psi as

$$E_c = w^{1.5}\, 33\sqrt{f'_c} \qquad (1)$$

where

$w \equiv$ unit weight of concrete, pcf

$f'_c \equiv$ compressive strength of concrete, psi

For normal weight concrete, E_c, may be taken as

$$E_c = 57{,}000\ \sqrt{f'} \qquad (2)$$

The maximum usable strain at the extreme concrete compression fiber shall be assumed equal to 0.003.

Tensile strength of concrete shall be neglected in flexural calculations of reinforced concrete.

The relation between the concrete compressive stress distribution and concrete strain, for sections subjected to flexure or flexure plus direct load, shall be assumed satisfied by an equivalent rectangular concrete stress distribution. The rectangular stress distribution is defined by the following. A concrete stress of 0.85 f'_c shall be assumed uniformly distributed over an equivalent compression zone bounded by the edges of the cross section and a straight line located parallel to the neutral axis at a distance

$$a = \beta_1 c \tag{3}$$

from the fiber of maximum compressive strain. The distance, c, from the fiber of maximum strain to the neutral axis shall be measured perpendicular to that axis. The factor, β_1, shall be taken as follows.

For $f'_c \leq 4000$

$$\beta_1 = 0.85 \tag{4}$$

For $4000 \leq f'_c \leq 6000$

$$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right) \tag{5}$$

The rectangular stress distribution does not duplicate the actual stress distribution in the compression zone at ultimate, but it does provide predictions of ultimate strength that are essentially the same as obtained from comprehensive tests.

Balanced strain conditions exist at a cross section when the tensile reinforcement farthest from the compression face, reaches the strain corresponding to its specified yield strength, f_y, just as the concrete in compression reaches its assumed ultimate strain of 0.003.

Load Factors

The service loads are multiplied by load factors to obtain the required strength. Two basic sets of load factors are given. Let the required strength be U, the service dead load be D, and the service combined load be C; where C is any combination of loads, except D, giving a maximum requirement. Then

$$U = 1.8D + 1.8C \tag{6}$$

or

$$U = 0.9D + 1.8C \tag{7}$$

Due regard must be given to sign in determining U for combinations of loadings, as one type of loading may produce effects of opposite sense to that produced by another type. The loading with 0.9D is specifically included for the case where dead load reduces the effects of other loads. Consideration must be given to various combinations of loadings to determine the most critical design condition. Note that some load producers generate both D and C loadings. For example, earth loading on a cantilever retaining wall generates lateral earth pressures as C loading and vertical earth weight as D loading.

Required strength may be expressed in several different ways. The designer has the choice of multiplying the service loads by the load factors before computing the factored load effects, or computing the effects of the unfactored loads and multiplying these effects by the load factors. For example, in the computation of moment for dead and combined load, repeating equation 7,

$$U = 0.9D + 1.8C \tag{7}$$

the designer may determine

$$w_u = 0.9w_d + 1.8w_c \tag{8}$$

where

w_u = factored uniform load

w_d = service uniform dead load

w_c = service uniform combined load

and then compute the factored moment, M_u, using w_u or, compute the dead and combined service moments and then determine the factored moment as

$$M_u = 0.9M_d + 1.8M_c \tag{9}$$

where

$M_u \equiv$ factored moment, or required strength in terms of moment

$M_d \equiv$ service dead load moment

$M_c \equiv$ service combined load moment.

Strength Reduction Factors

Nominal (or ideal) strengths are multiplied by strength reduction factors to obtain reasonably dependable or design strengths. The strength reduction factor, ϕ, represents an attempt to take account of several facets. These are: to allow for understrength due to variations in material strengths and dimensions; to allow for inaccuracies in design equations; to reflect the ductility and failure mode of the member under the type of loads under consideration; and to reflect the importance of the member in the structure.

Strength reduction factors, ϕ, shall be:

pure flexure	$\phi = 0.90$
direct tension	$\phi = 0.90$
flexure plus direct tension	$\phi = 0.90$
direct compression	$\phi = 0.70$
flexure plus direct compression except see below for low values of direct compression	$\phi = 0.70$
flexure plus low direct compression	ϕ = varies

let

$$P' \equiv \text{smaller of } 0.10\, f'_c\, A_g \text{ or } 0.7\, P_b \qquad (10)$$

then

$$\phi = 0.90 - 0.20\, P_u/P' \geq 0.70 \qquad (11)$$

where

$A_g \equiv$ gross area of section, sq. in.

$P_b \equiv$ nominal direct force strength at balanced strain condition

$P_u \equiv$ required direct force strength at given eccentricity

bearing on concrete	$\phi = 0.70$
beam shear	$\phi = 0.85$
torsional shear	$\phi = 0.85$

Flexure

General

Expressions for the nominal strength of rectangular and flanged members are obtained using the assumption of an equivalent rectangular stress distribution and the requirements of static equilibrium and compatibility of strains.

Rectangular Section with Tension Reinforcement Only

For rectangular sections, the nominal moment strength, M_n, may be computed as:

$$M_n = f_y A_s(d - a/2) \tag{12}$$

where

$$a = \frac{f_y A_s}{0.85 f'_c b} \tag{13}$$

in which

M_n ≡ nominal moment strength, inches-lbs
f_y ≡ yield strength of reinforcement, psi
A_s ≡ area of tension reinforcement, sq. in.
d ≡ effective depth of section, distance from extreme compression fiber to centroid of tension reinforcement, inches
a ≡ depth of equivalent rectangular stress block, inches
f'_c ≡ compressive strength of concrete, psi
b ≡ width of rectangular member, inches

The design moment strength is ϕM_n, and the required moment strength is M_u. Therefore the necessary nominal moment strength is

$$M_n = M_u/\phi \tag{14}$$

The steel ratio producing balanced strain conditions in these sections is

$$\rho_b = \bar{\rho}_b = \frac{A_s}{bd} = 0.85\, \beta_1 \frac{f'_c}{f_y}\left(\frac{87000}{87000 + f_y}\right) \tag{15}$$

$\bar{\rho}_b$ is defined here, for future reference, as the steel ratio producing balanced strain conditions in a rectangular section with tension steel only.

The maximum steel ratio permitted for these sections for <u>other structures</u> is one-half the steel ratio producing balanced strain conditions or

$$\rho_{max} = 0.50\, \bar{\rho}_b \tag{16}$$

Limiting the maximum steel ratio to this value ensures ductile behavior of flexure members. Failure, if it occurs, will be accompanied by large deflections giving ample warning of distress. This value is also sufficiently low to permit redistribution of moments in continuous members, see (8.4.1 - 8.4.3).

The Service has a good experience record with hydraulic structures designed by its working stress design criteria. This criteria has evolved over the years and represents a considerable experience investment. Designs based on balanced allowable working stresses result in relatively deep sections and low steel ratios as compared to strength design which tends to permit higher steel ratios based on balanced strain conditions. Deep sections imply small deflections and small cracks. Small cracks aid durability through increased resistance to deterioration. Therefore, the maximum steel ratio permitted for Service hydraulic structures is related to maximum steel ratios allowed by Service working stress design criteria and is given by

$$\rho_{shy} = 0.40 \frac{f'_c}{f_y}\left(\frac{1}{1 + \frac{1.25 f_y}{n f'_c}}\right) \tag{17}$$

in which $n = 503.3/\sqrt{f'_c}$ (18)

where

$n \equiv$ ratio of modulus of elasticity of steel to that of concrete.

Rectangular Sections with Compression Reinforcement

For rectangular sections, assuming the compressive steel reaches the yield strength, the nominal moment strength, M_n, may be computed by:

$$M_n = f_y(A_s - A'_s)(d - a/2) + f_y A'_s (d - d') \tag{19}$$

where

$$a = \frac{f_y (A_s - A'_s)}{0.85 f'_c b} \tag{20}$$

in which

A'_s = area of compression steel, sq. inches

d' = distance from extreme compression fiber to centroid of compression steel, inches.

Note that the equation for nominal moment strength does not include a correction for concrete displacement by the compression steel. Usually the correction is neglected. The correction can be included by substituting $(f_y - 0.85 f'_c)$ for f_y in the second term of the equation.

The steel ratio producing balanced strain conditions in these sections is:

$$\rho_b = \frac{A_s}{bd} = \bar{\rho}_b + \rho' \tag{21}$$

in which, by definition

$$\rho' = \frac{A'_s}{bd} \tag{22}$$

and, as defined earlier, $\bar{\rho}_b$ is the steel ratio producing balanced strain conditions in a rectangular section with tension steel only.

The compression steel and its associated tension steel do not contribute to brittle failure. They form a ductile auxiliary couple. Therefore dectile behavior is entirely ensured if the maximum steel ratio for <u>other structures</u> is limited so that:

$$\rho_{max} = 0.50\,\bar{\rho}_b + \rho' \tag{23}$$

or

$$(\rho - \rho')_{max} = 0.50\,\bar{\rho}_b \tag{24}$$

The maximum steel ratio for <u>Service hydraulic structures</u> is limited so that

$$(\rho - \rho')_{max} = \rho_{shy} \tag{25}$$

The assumption that the compressive steel reaches yield strength at nominal moment is only valid when

$$(\rho - \rho')_{min} = \frac{(A_s - A'_s)}{bd} \geq 0.85\,\beta_1 \frac{f'_c}{f_y} \frac{d'}{d} \left(\frac{87000}{87000 - f_y}\right) \tag{26}$$

or

$$d_{min} = 0.85\,\beta_1 \frac{f'_c}{f_y} d' \left(\frac{87000}{87000 - f_y}\right) \frac{1}{(\rho - \rho')} \tag{27}$$

When $(\rho - \rho')$ is less than the above amount, or when d is less than d_{min}, the compression steel stress is less than the yield strength. In this case, the effects of compression steel may be neglected and the nominal moment may be computed by the expression for rectangular sections with tension steel only. If this is done, the steel ratio limits for rectangular sections with tension steel only apply.

Compression steel may be used to increase load carrying capacity when a member is limited in cross section. However, strength design will show that this is not efficient use of steel. The use of compression steel does however have a marked effect on long-time deflections of flexural members.

Flanged Sections with Tension Reinforcement Only

If the depth of the equivalent rectangular stress block, a, does not exceed the thickness of the compression flange, t, then design may proceed using the relations for a rectangular section with tension steel only. The width, b, of the section is the distance out to out of the flange.

If the depth of the equivalent rectangular stress block, a, exceeds the compression flange thickness, t, then the nominal moment strength, M_n, may be computed by

$$M_n = f_y(A_s - A_{sf})(d - a/2) + f_y A_{sf} (d - t/2) \tag{28}$$

Where A_{sf} is the steel area necessary to develop the compressive strength of the overhanging flanges. It is given by

$$A_{sf} = 0.85 f'_c (b - b_w)t/f_y \tag{29}$$

and here

$$a = \frac{f_y(A_s - A_{sf})}{0.85 f'_c b_w} \tag{30}$$

where

- t ≡ compression flange thickness, inches
- b ≡ width of the compression flange, inches
- b_w ≡ width of the web, inches
- A_s ≡ total area of tensile reinforcement, sq. inches

The steel ratio producing balanced strain conditions in these sections is obtained by summing forces, or

$$f_y A_s = f_y(A_s - A_{sf}) + f_y A_{sf} \tag{31}$$

or

$$\rho_b bd = \overline{\rho}_b b_w d + \rho_f b_w d \tag{32}$$

so that

$$\rho_b = \frac{A_s}{bd} = \frac{b_w}{b} (\overline{\rho}_b + \rho_f) \tag{33}$$

Note that ρ_b is written as a function of b, but $\overline{\rho}_b$ and ρ_f are written as functions of b_w. $\overline{\rho}_b$ is the steel ratio producing balanced strain conditions in a rectangular section of width b_w with tension steel only.

The compression concrete of the overhanging flanges and its associated tension steel form an auxiliary couple which could contribute to brittle failure of the section. Therefore to ensure ductile behavior, the maximum steel ratio for other structures is limited to one-half the steel ratio producing balanced strain conditions.

That is

$$\rho_{max} = 0.50 \frac{b_w}{b} (\bar{\rho}_b + \rho_f) \tag{34}$$

or

$$(\rho - 0.50 \frac{b_w}{b} \rho_f)_{max} = 0.50 \frac{b_w}{b} \bar{\rho}_b \tag{35}$$

The maximum steel ratio for Service hydraulic structures is similarly limited. Thus

$$\rho_{max} = \frac{\rho_{shy}}{\bar{\rho}_b} \frac{b_w}{b} (\bar{\rho}_b + \rho_f) \tag{36}$$

or

$$\rho_{max} = \frac{b_w}{b} (\rho_{shy} + \frac{\rho_{shy}}{\bar{\rho}_b} \rho_f) \tag{37}$$

or

$$(\rho - \frac{\rho_{shy}}{\bar{\rho}_b} \frac{b_w}{b} \rho_f)_{max} = \frac{b_w}{b} \rho_{shy} \tag{38}$$

note that ρ_{shy} as used here is a function of b_w.

Flexure and Direct Force

The provisions herein are limited to rectangular sections and, in the case of compressive direct force, to short members. Since members are short, column slenderness effects are neglected. See (10.10 and 10.11) when slenderness effects must be included.

Design of members subject to flexure and direct forces is based on the equations of equilibrium and strain compatibility. The equivalent rectangular concrete compression stress block is assumed. Design strength must equal or exceed required strength. Therefore the necessary nominal moment strength and simultaneously necessary nominal direct force strength are both determined by dividing the required moment strength and the required direct force strength by the appropriate strength reduction factor, ϕ.

Interaction Diagrams

An interaction diagram is very helpful in describing, interpreting, and understanding the behavior of sections subjected to combined flexure and direct force. Direct forces are plotted as ordinates and moments as abscissas. Diagrams may be constructed in terms of design strengths or in terms of nominal strengths. Often the latter is preferred. Moments may be given as: moments about the mid-depth of the section (as usually determined from structural analysis), moments about the plastic centroid of the section (convenient since moments are zero at maximum compressive direct force), or moments about the centroid of the tension steel (instructive in understanding behavior). Care must be used to recognize or specify the reference moment center being used.

Thinking in terms of compressive direct forces and moments about the plastic centroid, any loading which plots within the enclosed area is a safe loading; any loading which plots on the curve represents a loading that will just fail the member; and any combination plotting outside the area is a failure combination. Any radial line from the origin represents a constant eccentricity of the load. Three points along the interaction curve are of significance. These are: (1) pure compression, loading is P_o; (2) balanced strain condition, loading is P_b and M_b; and (3) pure flexure, loading is M_o. The portion of the curve from P_o to P_b pertains to the range of eccentricities in which failure is initiated by crushing of the concrete. The portion of the curve from M_b to M_o pertains to the range of eccentricities in which failure is initiated by yielding of the tension steel.

Balanced Strain Condition

The balanced strain condition was defined previously. The distance from the extreme compression fiber to the axis of zero strain, c_b, is

$$c_b = \left(\frac{87000}{87000 + f_y}\right)d \tag{39}$$

The depth of the stress block, a_b, is

$$a_b = \quad c_b = \quad \left(\frac{87000}{87000 + f_y}\right)d \tag{40}$$

With reinforcement in one or two faces, each parallel to the axis of bending and all the steel in any one face located at closely the same distance from the axis of bending, the balanced nominal compressive direct force strength, P_b, is:

$$P_b = 0.85\ f'_c\ a_b\ b + f_y\ A'_s - f_y\ A_s \tag{41}$$

This expression does not include a correction for concrete displaced by the compression steel. A correction may be included as indicated previously, if felt desirable. Also the expression as written assumes the compression steel is at yield strength. If the stress in the compression steel is less than yield strength, then either, the compression steel may be neglected, or the stress in the compression steel may be evaluated by strain proportionality relations and the expression modified accordingly.

The balanced nominal moment strength, M_b, moments taken about the plastic centroid is:

$$M_b = P_b\ e_b = 0.85\ f'_c\ a_b\ b\ (d - d''' - a_b/2) + f_y\ A'_s\ (d - d' - d''') + f_y\ A_s\ d''' \tag{42}$$

where

$e_b \equiv$ eccentricity, at balanced strain conditions, of the direct force measured from the plastic centroid of the section, inches

$d''' \equiv$ distance from plastic centroid of section to centroid of tension reinforcement, inches

Failure Controlled by Tension

For the same section defined above for balanced strain conditions, the nominal compressive direct force strength, P_n, is

$$P_n = 0.85\ f'_c\ ab + f_y\ A'_s - f_y\ A_s \tag{43}$$

and the nominal moment strength, M_n, moments taken about the centroid of the tension steel, is:

$$M_n = P_n\ z = 0.85\ f'_c\ ab(d - a/2) + f_y\ A'_s(d - d') \tag{44}$$

where

$z \equiv$ eccentricity of the direct force measured from the centroid of the tensile reinforcement, inches.

Usual comments about correction for displaced concrete apply. Also strain compatibility calculations are required to check that the compression steel will actually yield at the nominal moment strength of the section.

The direct force equation may be used to obtain the depth of the stress block, a, as:

$$a = \frac{P_n + f_y A_s - f_y A'_s}{0.85\, f'_c\, b} \tag{45}$$

With the depth, a, known, the moment strength may be determined.

The concept indicated above for sections controlled by tension is quite general and may always be used. An alternate formulation is possible however which simplifies design for combined flexure and direct force.

Transfer the force system on the section from that of a direct force acting at mid-depth and a moment taken about the mid-depth to a direct force acting along the centroidal axis of the tensile steel and a moment about the centroid of the tensile steel. If the nominal moment strength about the mid-depth is M_n, then the resultant equivalent nominal moment strength about the tension steel, M_{ns}, is

$$M_{ns} = M_n + P_n\, d'' \tag{46}$$

where

$d'' \equiv$ distance from mid-depth of section to centroid of tension reinforcement, inches.

A section loaded with the equivalent nominal moment strength, M_{ns}, and reinforced with an equivalent tensile steel area, A, where

$$A = A_s + P_n/f_y \tag{47}$$

will have the same compressive concrete stress block and same concrete and steel strains as a section reinforced with tensile steel, A_s, and loaded with the nominal moment strength, M_n, and nominal direct force strength, P_n. Therefore the required tensile steel area, A_s, may be determined as

$$A_s = A - P_n/f_y \tag{48}$$

where

$A \equiv$ equivalent tensile steel area required by flexure only due to the moment, M_{ns}, sq. inches.

In the above relations, compressive direct force is positive and tensile direct force is negative. The only restrictions on the formulation are that neither A_s nor M_{ns} may be negative quantities.

The maximum steel ratio for sections controlled by tension for other structures is limited so that

$$(\rho - \rho')_{max} = 0.50\ \bar{\rho}_b \tag{49}$$

The maximum steel ratio for sections controlled by tension for Service hydraulic structures is limited so that

$$(\rho - \rho')_{max} = \rho_{shy} \tag{50}$$

When sections are subjected to combined flexure and low compressive direct force, failure is controlled by yielding of the tension steel. The value of the strength reduction factor varies and depends on the cross section and material properties. The design procedure is not direct. Some form of guess and check approach is necessary. One approach is to assume $\phi = 0.70$ for all combinations of flexure and direct compressive force. This is conservative but penalizes design as the compressive direct force approaches zero. An alternate approach is to assume an initial $\phi = 0.70$, obtain the resulting section, recompute ϕ which might exceed 0.70, obtain a new, reduced, trial section, and recycle the steps as necessary until a stable design is reached.

Failure Controlled by Compression

Statics and strain compatibility relations may be used to determine nominal moment strength and nominal compressive direct force strength for sections when failure is controlled by compression of the concrete. The shape of the interaction diagram in the region between the maximum possible nominal compressive direct force strength, P_o, and the nominal compressive direct force strength at balanced strain conditions, P_b, may be defined by the determination of a sufficient number of nominal load combinations.

As an alternate to the procedure indicated above, the nominal compressive direct force strength, P_n, may be assumed to decrease linearly from P_o to P_b as the nominal moment strength increases from zero to M_b. The nominal compressive direct force strength at zero eccentricity, P_o, is

$$P_o = 0.85\ f'_c\ (A_g - A_{st}) + f_y\ A_{st} \tag{51}$$

where

$A_g \equiv$ gross area of section, sq. inches

$A_{st} \equiv$ total steel area of section, sq. inches

Thus P_n for an associated nominal moment strength, M_n, the moment taken about the plastic centroid, is

$$P_n = P_o - (P_o - P_b)(M_n/M_b) \tag{52}$$

The maximum usable nominal compressive direct force strength for rectangular sections is arbitrarily limited to

$$P_{n(max)} = 0.80\ P_o \tag{53}$$

The limitation is intended to account for accidental eccentricities on the section that are not accounted for in the design analysis.

Beam Shear

Flexural, or beam, shear and torsional shear are intimately related. Nominal shear strength provided by concrete and nominal torsional moment strength provided by concrete are functions one of the other. Requirements for shear reinforcement and requirements for torsion reinforcement are additive. For convenience, beam shear is presented first.

The relatively abrupt nature of a shear failure in a member with an unreinforced web, as compared to a ductile flexural failure, makes it desirable to design members so that strength in shear is at least as great as strength in flexure. To ensure that failure is by ductile flexural mode, the Code limits, the maximum amount of longitudinal reinforcement and, except for certain types of structural components, requires that at least a minimum amount of web reinforcement be provided in all flexural members.

Most so-called shear failures are really diagonal tension failures. Computed shear stresses are really just stress indices, only slightly related to actual stresses.

Shear strength is therefore based on the average shear stress on the full effective cross section, $b_w d$, where b_w is the web width of the member. Permissible shear stresses are functions of $\sqrt{f'_c}$. Units of $\sqrt{f'_c}$ are psi, where f'_c is in psi.

The provisions herein only highlight the Code requirements. The Code should be referenced for completeness.

Shear Strength

The design shear strength, ϕV_n, must equal or exceed the required shear strength V_u, at the section under consideration. Therefore necessary nominal shear strength, V_n, is given by

$$V_n = V_u/\phi \tag{54}$$

The nominal shear strength, V_n, includes the nominal shear strength provided by the concrete, V_c, and the nominal shear strength provided by the shear reinforcement, V_s, or

$$V_n = V_c + V_s \tag{55}$$

When the reaction introduces compression into the end regions of a member, shear strength is increased. Accordingly, design may be based on the maximum required shear strength, V_u, at a distance, d, from the face of the support. For many other situations, the critical section for shear should be taken at the face of the support.

Shear Strength Provided by Concrete

More detailed relations are available from which the nominal shear strength provided by the concrete may be computed; however for most designs, the following expressions are convenient and satisfactory. For members subject to shear and flexure only:

$$V_c = 2 \sqrt{f'_c}\, b_w d \tag{56}$$

For members subject to direct compression in addition to shear and flexure:

$$V_c = 2\left(1 + \frac{N_u}{2000\, A_g}\right) \sqrt{f'_c}\, b_w d \tag{57}$$

where

N_u ≡ required compressive direct force strength acting simultaneously with V_u, taken positive for compression, lbs.

For members subject to direct tension in addition to shear and flexure, web reinforcement should be designed to carry the total shear unless an analysis is made using

$$V_c = 2\left(1 + \frac{N_u}{500\, A_g}\right) \sqrt{f'_c}\, b_w d \tag{58}$$

where

N_u ≡ required tensile direct force strength acting simultaneously with V_u, taken negative for tension, lbs.

Shear strength that can be provided by concrete decreases with the amount of torsion present at a section. The effect of torsion can be neglected if the required torsional moment strength, T_u, is less than

$$T_u \leq \phi\, 1.5 \sqrt{f'_c}\, \frac{1}{3} \Sigma\, (x^2 y) \tag{59}$$

where

T_u ≡ required torsional moment strength at section considered, inches-lbs

x ≡ shorter dimension of a rectangular component of the cross section, inches

y ≡ longer dimension of a rectangular component of the cross section, inches

If the required torsional moment strength, T_u, is greater than the above amount, then the nominal concrete shear strength, V_c, available is a function of an interaction relation and is given by

$$V_c = \frac{2\sqrt{f'_c}\, b_w d}{\sqrt{1 + \left(2.5\, C_t \frac{T_u}{V_u}\right)^2}} \tag{60}$$

where

$C_t = \dfrac{b_w d}{\Sigma(x^2 y)}$, a factor relating torsion and shear forces to torsion and shear stresses.

Shear Strength Provided by Shear Reinforcement

Shear reinforcement restrains the growth of inclined cracking. It therefore increases ductility and provides a warning of distress. A minimum area of shear reinforcement is required in all flexural members wherever the required shear strength, V_u, exceeds one-half the design shear strength provided by the concrete, ϕV_c. Three types of members are excluded from the minimum shear reinforcement requirement. These are slabs and footings, floor joists, and relatively wide, shallow beams. Where shear reinforcement is required, either by strength computations or by provision of the minimum amount, and torsion is negligible in accordance with equation 59, the minimum shear reinforcement area is

$$A_v = 50\, b_w s/f_y \tag{61}$$

where

$s \equiv$ spacing of shear reinforcement in the direction parallel to the longitudinal reinforcement, inches

$A_v \equiv$ area of shear reinforcement within the distance, s, for example, area of two legs when closed stirrups are used, sq. inches.

Where shear reinforcement is required and torsion is not negligible, the minimum area of closed stirrups is

$$A_v + 2A_t = 50\, b_w s/f_y \tag{62}$$

where

$A_t \equiv$ area of one leg of a closed stirrup resisting torsion within the distance, s, sq. inches.

Shear reinforcement is required whenever the necessary nominal shear strength, V_n, exceeds the nominal shear strength, V_c, provided by the concrete. When shear reinforcement is provided by vertical stirrups, (stirrups placed perpendicular to the member axis), the nominal shear strength provided by shear reinforcement, V_s, is

$$V_s = \frac{A_v f_y d}{s} \tag{63}$$

or

$$A_v = \frac{(V_u/\phi - V_c)s}{f_y d} \tag{64}$$

Spacing of vertical stirrups may not exceed the distance, d/2. Whenever V_s exceeds $4 \sqrt{f'_c}\, b_w d$, the maximum spacing of vertical stirrups is decreased to the distance, d/4. Nominal shear strength provided by shear reinforcement, V_s, shall not exceed $8 \sqrt{f'_c}\, b_w d$.

Torsion

Torsion can be important in members acting as spandrel beams, in curved beams, and wherever members carry transverse loads eccentric to the member axis. Elastic theory for members with circular cross sections, results in tangential shearing stresses varying with distance from the center of the circle. These stresses are given by

$$v_t = Tr/J \tag{65}$$

where

$v_t \equiv$ torsional shear stress

$T \equiv$ torque or torsional moment

$r \equiv$ distance from centroid of section

$J \equiv$ polar moment of inertia of the section

Plastic theory for members with circular cross sections results in tangential shearing stresses which are uniform throughout the cross section.

Elastic theory for single rectangular sections, shows maximum torsional shear stress occurs at the center of the long side and parallel to it. For sections consisting of multiple rectangles, the maximum shear occurs at the center of the long side of the rectangle having the greatest thickness. Neither elastic nor plastic theory is directly applicable to reinforced concrete because of the nature of the material and because the concrete will be cracked at significant loading.

An approximate stress relation is used for reinforced concrete. The relation applies to cross sections consisting of either single or multiple rectangles. The relation is

$$v_t = \frac{T}{\frac{1}{3}\,\Sigma(x^2y)} \tag{66}$$

where

$x \equiv$ shorter dimension of a rectangular component of the cross section, inches

$y \equiv$ longer dimension of a rectangular component of the cross section, inches

The calculation of $\Sigma(x^2y)$ depends on the selection of the component rectangles. The rectangles may not overlap. They may be taken so as to result in the maximum possible summation.

The provisions herein only highlight the Code requirements. The Code should be referenced for completeness.

Torsion effects may be neglected when the maximum torsional stress does not exceed a limiting value of 1.5 $\sqrt{f'_c}$ psi. This stress corresponds to approximately one-fourth the pure torsional strength of a member without torsional reinforcement. Thus in terms of required torsional moment strength, T_u, torsion may be neglected if:

$$T_u \leq \phi\ 1.5\ \sqrt{f'_c}\ \frac{1}{3}\ \Sigma(x^2y) \tag{67}$$

where all terms have been defined previously. If torsion exceeds the above amount, then torsion effects must be included with beam shear and flexure.

In designing for torsion, two states of structural behavior should be recognized, based upon whether or not redistribution of internal forces is possible. If the required torsional moment strength, T_u, is required to maintain equilibrium, as in a statically determinate structure, then the torsional moment cannot be reduced by redistribution of internal forces. In this case the member must be designed for the full required torsional moment strength. This state of torsion is referred to as "equilibrium torsion."

In a statically indeterminate structure, redistribution of internal forces will occur as the loading reaches and exceeds the loading corresponding to some limiting strength value in the structure. Therefore, when torsional moments exist in a statically indeterminate structure, the magnitude of the torsional moment is dependent on whether or not redistribution of loads between the member under consideration and the remaining interacting elements occurs. If the computed elastic torsional moment before redistribution is greater than

$$T_n = T_u/\phi = 4\ \sqrt{f'_c}\ \frac{1}{3}\ \Sigma(x^2y) \tag{68}$$

then torsional cracking is assumed. At torsional cracking, a large twist occurs under essentially constant torque. A significant redistribution of forces within the structure results. This state of torsion is referred to as "compatibility torsion." Thus, when the computed elastic torsional moment exceeds the cracking torque, redistribution is assumed. A maximum necessary nominal torsional moment strength, taken equal to the cracking torque given by equation 68, may be assumed at the critical sections. This reduced torsional moment must then be used to determine adjusted shears and moments in the adjoining structural elements.

If, in a statically indeterminate structure, the torsional moment calculated by elastic analysis is less than the cracking moment given by equation 68, then torsional cracking and hence redistribution will not occur. In this case, the actual computed torsional moment should be used in design.

Torsional Moment Strength

The design torsional moment strength, ϕT_n, must equal or exceed the required torsional moment strength, T_u, at the section under consideration. Therefore necessary nominal torsional moment strength, T_n, is given by

$$T_n = T_u/\phi \qquad (69)$$

The nominal torsional moment strength, T_n, includes the nominal torsional moment strength provided by the concrete, T_c, and the nominal torsional moment strength provided by the torsion reinforcement, T_s, or

$$T_n = T_c + T_s \qquad (70)$$

Sections located less than a distance, d, from the face of a support may be designed for the same torsional moment as that computed at a distance, d.

Torsional Moment Strength Provided by Concrete

Torsional moment strength provided by concrete depends on the beam shear present at the section. The interaction relation is

$$T_c = \frac{2.4\sqrt{f'_c}\,\frac{1}{3}\,\Sigma(x^2y)}{\sqrt{1 + \left(\frac{V_u}{2.5\,C_t\,T_u}\right)^2}} \qquad (71)$$

For members subject to significant direct tension, torsion reinforcement should be designed to carry the total torque unless an analysis is made in which T_c given by equation 71, and V_c given by equation 60, are both multiplied by the factor $(1 + N_u/500\,A_g)$ of equation 58, where N_u is negative for direct tension.

Torsional Moment Strength Provided by Torsion Reinforcement

Reinforcement required to resist torsion is added to that required to resist shear, flexure, and direct forces. Both closed transverse reinforcement and longitudinal reinforcement are required to resist the diagonal tension stresses due to torsion. Stirrups must be closed since inclined torsional cracking, depending on cross section proportions, may appear on any face of a member. Torsional diagonal cracks develop in a helical pattern, therefore torsion reinforcement should be provided at least a distance, (d + b) beyond the theoretically required point.

Spacing of closed stirrups shall not exceed the smaller of $(x_1 + y_1)/4$ or 12 inches, where

x_1 ≡ shorter center-to-center distance of the closed rectangular stirrup under consideration, inches

y_1 ≡ longer center-to-center distance of the closed rectangular stirrup under consideration, inches

Spacing of longitudinal reinforcement, distributed around the inside perimeter of the closed stirrups, shall not exceed 12 inches. At least one longitudinal bar shall be placed in each corner of the closed stirrups.

Torsion reinforcement is required whenever the necessary nominal torsional moment strength, T_n, exceeds the nominal torsional moment strength, T_c, provided by the concrete. Nominal torsional moment strength provided by torsion reinforcement, T_s, is computed by

$$T_s = \frac{A_t \alpha_t x_1 y_1 f_y}{s} \tag{72}$$

or

$$A_t = \frac{(T_u/\phi - T_c)s}{f_y \alpha_y x_1 y_1} \tag{73}$$

where

$$\alpha_t = 0.66 + 0.33 y_1/x_1 \leq 1.50 \tag{74}$$

and

A_t = area of one leg of a closed stirrup resisting torsion within the distance, s, sq. inches

s ≡ spacing of torsion reinforcement in the direction parallel to the longitudinal reinforcement, inches.

Where torsion reinforcement is required, the minimum area of closed stirrups is given by equation 62.

In flanged sections, closed stirrups may be placed in either the largest or in all component rectangles. In the first, x and y refer to the stirrup dimensions in the largest rectangle. In the second case, equations 72, 73, and 74 may be applied separately to each component rectangle using x_1 and y_1 for the rectangle under consideration. The nominal torsional moment strength provided by the torsion reinforcement in the component rectangles must sum to the total T_s needed. Thus,

$$T_s = \Sigma\left(\frac{A_t \alpha_t x_1 y_1 f_y}{s}\right) \tag{75}$$

If all stirrups are the same bar size and spacing, then

$$T_s = \frac{A_t}{s} f_y \; \Sigma(\alpha_t x_1 y_1) \tag{76}$$

or

$$\frac{A_t}{s} = \frac{(T_u/\phi - T_c)}{f_y \Sigma(\alpha_t x_1 y_1)} \tag{77}$$

As members reach their maximum torsional strength, additional stress moves into the longitudinal steel. A torsion stirrup cannot develop its full strength unless sufficient longitudinal steel is present. The action may be visualized as a kind of truss system where the longitudinal steel provides the tension chord while the stirrup provides the tension vertical.

Required area of longitudinal bars, A_ℓ, distributed around the perimeter of the closed stirrup corresponding to A_t, is given by the larger of

$$A_\ell = 2 A_t (\frac{x_1 + y_1}{s}) \tag{78}$$

or

$$A_\ell = \left[\frac{400\ xs}{f_y} \left(\frac{T_u}{T_u + \frac{V_u}{3 C_t}}\right) - \zeta\right] (\frac{x_1 + y_1}{s}) \tag{79}$$

where ζ is the larger of $2 A_t$ or $50 b_w s/f_y$ and x is the shorter side of the component rectangle under consideration. The first relation sets the volume of the longitudinal reinforcement in the distance s equal to the volume of the closed stirrup. The second relation recognizes that required longitudinal steel decreases with the amount of closed transverse steel provided and also decreases with beam shear.

The provisions for torsion reinforcement design depend on the development of yield stress in torsion reinforcement prior to ultimate load, that is, bebore the compression concrete crushes. Pure torsion tests indicate that to ensure torsion reinforcement yielding, the nominal torsional stress should not exceed $12 \sqrt{f'_c}$. This translates, for torsion combined with beam shear and flexure, when written in terms of torsional moment, to

$$T_n \leq 5 T_c \tag{80}$$

or

$$T_s \leq 4 T_c \tag{81}$$

Control of Deflections

To serve its intended function a structure must be safe and serviceable throughout its design life. Serviceability and durability require that deflections and attendant cracking be kept unobjectionably small. With strength design and the use of high strength steels, there is a tendency toward shallow sections. Shallow sections produce large deflections. Hence deflections assume increased importance in strength design procedures.

The deflections of interest are those that occur at service load levels. They may be deflections that occur immediately on application of load or they may be long-time deflections caused by shrinkage and by creep under sustained load. Under service loads the steel and concrete stresses are essentially within their elastic ranges so that deflections that occur immediately may be calculated by methods based on elastic behavior.

Two approaches are provided for controlling deflections. Deflection requirements are considered satisfied if at least a specified minimum thickness for the type of member is used. For members that do not meet the minimum thickness criteria, deflections must be calculated.

Minimum Thicknesses

For one-way slabs, for two-way slabs having a ratio of long to short span exceeding two, and for beams, the minimum thicknesses, unless deflections are computed, shall be:

(1) for simply supported spans
  for slabs $\ell/20$, for beams $\ell/16$

(2) for one end continuous -
  for slabs $\ell/24$, for beams $\ell/18.5$

(3) for both ends continuous -
  for slabs $\ell/28$, for beams $\ell/21$

(4) for cantilever spans -
  for slabs $\ell/10$, for beams $\ell/8$.

Note that both thickness and span length are in the same units. The aforementioned two-way slabs act essentially as one-way, hence the reference span length, ℓ, is the short span length. For $f_y < 60000$ psi, the above values may be multiplied by $(0.4 + f_y/100{,}000)$.

For two-way slabs having a ratio of long to short span not exceeding two, the minimum thickness, h, shall be given in inches, by:

$$h = \ell_n(800 + 0.005\, f_y)/36000 \tag{82}$$

where

$\ell_n \equiv$ clear span in long direction, inches

but h need not exceed minimum thickness required as a one-way slab.

Deflection Computations

When deflections are computed, deflections that occur immediately on application of load shall be computed by usual methods for elastic displacements. Immediate deflections are sensitive to the amount of cracking along the span. For prismatic members, the effective moment of inertia, I_e, shall be computed by:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g \tag{83}$$

in which

$$M_{cr} = \frac{f_r\, I_g}{y_t} \tag{84}$$

and

$$f_r = 7.5\,\sqrt{f'_c} \tag{85}$$

where:

$I_g \equiv$ moment of inertia of gross concrete section about centroid axis and neglecting reinforcement, inches^4.

$I_{cr} \equiv$ moment of inertia of cracked section, transformed to concrete, inches^4.

$M_{cr} \equiv$ cracking moment, in-lbs.

$M_a \equiv$ maximum moment in member, or portion of member, for loading for which deflection is computed, in-lbs.

$f_r \equiv$ modulus of rupture of concrete, psi.

$y_t \equiv$ distance from centroidal axis of gross section, neglecting reinforcement, to extreme fiber in tension, inches.

Note that I_e cannot be taken greater than I_g. The modulus of elasticity of concrete, E_c, is given by equation 1 or 2.

For prismatic continuous spans, the effective moment of inertia may be taken as the average of values obtained from equation 80 for the critical positive and negative moment sections. Alternately, the effective moment of inertia may be obtained by weighing the positive and negative moment

I_e values in proportion to the lengths of the span in positive moment and in negative moment.

For nonprismatic members, that is, members whose depth or other dimension varies along the span, effective moments of inertia may be computed for critical locations or discrete distances as necessary to define the variation across the span.

Shrinkage and creep due to those loads that are sustained for long durations cause deflections to increase over time. The additional long-time deflection (in addition to the deflection that occurs immediately on application of load) caused by that portion of the load that is sustained, is obtained by multiplying the immediate deflection, caused by the sustained loads, by a factor, λ , given by

$$\lambda = (2 - 1.2\ A_s'/A_s) \geq 0.6 \tag{86}$$

The long-time deflections thus calculated are those that might be expected after loads are sustained for about three years. Note that the multiplication factor indicates the effectiveness of compression steel in reducing long-time deflections.

Unless it is determined that the larger displacements associated with lesser thicknesses will not cause adverse structural or servicability effects, the following limit shall apply. The maximum deflection under any combination of immediate or sustained loading, shall not exceed $\ell/240$.

Control of Flexural Cracking

Large crack widths are unsightly and result in poor resistance to corrosion. To assure protection of the reinforcement against corrosion, many fine cracks are preferable to few wide cracks. When reinforcing is used at high service load stresses, excessive flexural crack widths may be expected unless adequate precautions are taken in detailing the reinforcement.

Size and spacing of flexural cracks are functions of steel stress at service load levels, amount of concrete cover, and the area of concrete surrounding and tributary to each individual reinforcing bar. The importance of controlling cracking increases with the exposure or environment class of the structural component.

Cross sections at both maximum positive and maximum negative moment locations, in beams and one-way slabs, shall be proportioned so that the quantity, Z, given by,

$$Z = f_s \sqrt[3]{d_c A} \qquad (87)$$

does not exceed 130 for Service hydraulic structures, nor 145 for other structures. For information, maximum values of Z of 95 and 115 are commonly specified for the design of waste treatment works requiring long service life. In the expression of Z:

- f_s ≡ calculated stress in reinforcement at service loads, ksi
- d_c ≡ thickness of concrete cover measured from the extreme tension fiber to the center of the longitudinal bar located closest to the extreme fiber, inches
- A ≡ effective tension area of concrete per bar; determined as the tension area of concrete surrounding the flexural tension reinforcement and having the same centroid as the reinforcement, divided by the number of bars, sq. inches.

The calculated stress, f_s, in the reinforcement at service load shall be computed assuming elastic (working stress) action. In lieu of such calculations, the value of f_s may be taken as 0.60 f_y.

The maximum bar spacing, s, for one-way slabs with tensile reinforcement in a single layer at the section, is given in inches by

$$s = \frac{(Z/f_s)^3}{2\, d_c^{\,2}} \qquad (88)$$

Note that the permissible spacing decreases as concrete cover increases, as steel stress at service loads increases, and as environment class becomes more severe.

Flexural cracking behavior of two-way slabs, with ratio of long to short span not exceeding two, is significantly different from that in one-way members. For the same total load, the crack widths in two-way action will usually be less than those experienced in one-way action. The Code is silent on crack width relations in two-way slab systems, thus steel spacing is controlled by strength requirements, temperature and shrinkage requirements, or limitations on maximum spacing of reinforcement.

Development of Reinforcement

The calculated tension or compression in any bar at any section must be developed on each side of that section by proper embedment length, end anchorage, or hooks. Hooks may be used in developing bars in tension. Hooks are not effective in developing bars in compression.

The strength reduction factor is not used in development computations. The specified development lengths already include an allowance for understrength.

The development length concept is based on the attainable average bond resistance over the length of embedment of the reinforcement. If a reinforcing bar in a member has enough embedment in concrete, it cannot be pulled out of the concrete before the bar fails by yielding of the steel.

Flexural bond stresses are not addressed directly. Limitations pertaining to bar sizes permitted in positive moment reinforcement at simple supports and at points of inflection actually reflect consideration of flexural bond and required steel perimeter at these locations.

Development Lengths

Required development length, ℓ_d, in inches, is the product of the basic development length and applicable multipliers. The basic development length, ℓ_b, for deformed bars in tension, for bars #11 and smaller, is the larger of $0.04\ A_b\ f_y/\sqrt{f'_c}$ or $0.0004\ d_b\ f_y$ where

$A_b \equiv$ area of an individual bar, sq. inches
$d_b \equiv$ nominal diameter of a bar, inches.

Multipliers of bars in tension are:

Top reinforcement	1.4
Bars spaced laterally not less than 6 inches on center, and bars with at least 3 inches clear from face of member to first bar . .	0.8
When excess longitudinal reinforcement is provided	$\dfrac{A_s \text{ required}}{A_s \text{ provided}}$

Further, the development length, ℓ_d, for tension bars shall not be less than 12 inches, except in computations of either lap splice lengths or development lengths of web reinforcement. Top reinforcement is horizontal reinforcement so placed that more than 12 inches of concrete is cast in the member below the reinforcement.

The basic development length, ℓ_b, for deformed bars in compression is the larger of $0.02\ d_b\ f_y/\sqrt{f'_c}$ or $0.0003\ d_b\ f_y$. The multiplier for excess longitudinal reinforcement given above for tension bars also applies to compression bars. The development length, ℓ_d, for compression bars shall not be less than 8 inches.

Standard hooks in tension may be considered to develop a maximum tensile stress, f_h, at the point of tangency given by

$$f_h = \xi\sqrt{f'_c} \tag{89}$$

where

ξ ≡ factor for standard hook, given by Table 12.5.1 of the Code.

An equivalent embedment length, ℓ_e, of a standard hook may be computed as the larger of $0.04\ A_b\ f_h/\sqrt{f'_c}$ or $0.0004\ d_b\ f_h$ times any applicable multipliers.

The development length, ℓ_d, in tension may consist of a combination of equivalent embedment length, ℓ_e, plus additional embedment length of reinforcement $(\ell_d - \ell_e)$. This additional length must be provided between the point of tangency of the hook and the critical section, that is, the additional embedment length may not be bar extension beyond the standard hook.

Development of Flexural Reinforcement

Critical sections for development of reinforcement are at points of maximum stress and at points where adjacent reinforcement terminates.

Except at supports of simple spans and at the free end of cantilevers, every reinforcing bar shall be extended beyond the point at which it is no longer needed to resist flexural stress, for a distance equal to the effective depth of the member or 12 bar diameters, whichever is greater.

Continuing reinforcement shall have an embedment length not less than the development length, ℓ_d, beyond the point where bent or terminated tension reinforcement is no longer needed to resist flexural stress.

Flexural reinforcement shall not be terminated in a tension zone unless one of the following conditions is satisfied:

(*i*) Shear at the cutoff point does not exceed two-thirds of that permitted.

(*ii*) Stirrup area in excess of that required for shear and torsion is provided along each terminated bar over a distance from the termination point equal to three-fourths the effective depth. The excess stirrup area shall not be less than $60\ b_w s/f_y$. The spacing, s, shall not exceed $(d/8)/\beta_b$ where β_b is the ratio of area terminated to total area of tension reinforcement at the section.

(*iii*) For #11 bars and smaller, continuing reinforcement provides at least double the area required for flexure at the cutoff point and shear does not exceed three-fourths that permitted.

At least one-third the positive moment reinforcement in simple spans and one-fourth the positive moment reinforcement in continuous spans shall extend along the same face of the span into the support at least 6 inches.

At simple supports and at points of inflection, positive moment tension reinforcement shall be limited to a bar diameter such that the development length, ℓ_d, satisfies the relation

$$\ell_d \leq \frac{M_n}{V_u} + \ell_a \tag{90}$$

where

M_n ≡ nominal moment strength assuming all reinforcement at the section to be stressed to f_y, in-lbs

V_u ≡ required shear strength at the section, lbs

ℓ_a ≡ at a support, the sum of the embedment length beyond the center of the support and the equivalent embedment length of any hook; at a point of inflection, the effective depth of the member or 12 bar diameters, whichever is greater.

The value of M_n/V_u may be increased 30 percent when the ends of reinforcement are confined by a compressive reaction.

At least one-third the negative moment reinforcement at a support shall extend beyond the extreme position of the point of inflection a distance not less than the effective depth of the member, 12 bar diameters, or one-sixteenth the clear span, whichever is greater.

Development of Web Reinforcement

Ends of single leg, simple or multiple U-stirrups shall be anchored by one of the following means:

(*i*) A standard hook plus an embedment of 0.5 ℓ_d. The 0.5 ℓ_d embedment of a stirrup leg shall be taken as the distance between middepth of a member and the point of tangency of the hook.

(*ii*) Embedment d/2 above or below middepth on the compression side of the member for a full development length, ℓ_d, but not less than 24 bar diameters, or 12 inches.

(*iii*) For #5 bar and smaller, bending around a longitudinal bar through at least 135 degrees plus, for stirrups with a design stress greater than 40,000 psi, an embedment of 0.33 ℓ_d. The 0.33 ℓ_d embedment shall be taken as the distance between middepth of the member and the point of tangency of the hook.

Between anchored ends, each bend in the continuous portion of a stirrup shall enclose a longitudinal bar.

Pairs of U-stirrups so placed as to form a closed unit shall be considered properly spliced when length of laps are 1.7 ℓ_d. In members at least 18 inches deep, such stirrups with $A_b\, f_y \leq 9000$ lbs per leg may be considered adequately spliced if the stirrup legs extend the full available depth of member.

Lap Splices in Reinforcement

General

To ensure ductile behavior, lap splices should be adequate to develop more than the yield strength of the reinforcement. Splices should, if possible, be located away from points of maximum tensile stress. Splices should be made at or close to points of inflection if it is practical to do so.

Lap splices shall not be used for bars larger than #11. Lap splices need not be in contact. Bars in a noncontact splice shall not be farther apart than 1/5 the required length of lap nor 6 inches.

Lap splice lengths are in terms of the full yield stress of the bar. That is, required lengths are not functions of computed stress in the bar.

Tension Lap Splices

Three classes of tension lap splices are established. The minimum length of lap is determined as a multiplier for the class times the development length, ℓ_d, but not less than 30 bar diameters. The classes and minimum lengths are:

Class A splice . . . . 1.0 ℓ_d
Class B splice . . . . 1.3 ℓ_d
Class C splice . . . . 1.7 ℓ_d.

The splice class required depends upon the stress level in the reinforcement to be spliced and the portion of the total reinforcement to be spliced at the cross section.

If the area of tensile steel provided at the splice location is equal to or more than twice that required by analysis (low tensile stress in the reinforcement) and not more than 75 percent of the bars are to be lap spliced within the required lap splice length, a Class A splice may be used. If more than 75 percent of the bars are to be lap spliced within the required lap splice length, a Class B splice is required.

If the area of tensile steel provided at the splice location is less than twice that required by analysis (high tensile stress in reinforcement) and not more than 50 percent of the bars are to be lap spliced within the required lap splice length, a Class B splice may be used. If more than 50 percent of the bars are to be lap spliced within the required lap splice length, a Class C splice is required.

Note that the preferred splice layout thus consists of staggered splices all located away from sections of maximum tensile stress.

The development length, ℓ_d, used in the calculation of splice lap must incorporate the multiplier for top bars if applicable, may incorporate the multiplier for wide spacing when applicable, and may not incorporate the multiplier for excess longitudinal reinforcement. The effect of excess longitudinal reinforcement is included in determining the splice class to be used.

<u>Compression Lap Splices</u>

The minimum length of lap for compression lap splices is the larger of the development length, ℓ_d, in compression, 0.005 f_y d_b, 24 bar diameters, or 12 inches. For $f'_c < 3000$ psi, the lap length shall be increased by 1/3.

<u>Special Requirements - Flexure Plus Compressive Direct Force</u>

Special requirements are imposed when a section is subjected to combined flexure and direct compressive force. The interaction diagram aids in visualizing the requirements.

Any section where splices are located, a minimum tensile strength shall be provided in each face. The minimum strength shall equal at least 1/4 the area of longitudinal reinforcement in that face multiplier by the yield strength, f_y.

For conditions where the steel stress in a tension face varies from zero to 1/2 f_y, a minimum tensile strength at least equal to twice the calculated tension must be maintained in that face. The tensile strength may be provided by a combination of spliced bars and continuing unspliced bars, taken at the yield strength, f_y.

For conditions where the tensile steel stress exceeds 1/2 f_y, splice requirements are the same as those for a tension lap splice.

Details of Reinforcement

Concrete Cover for Reinforcement

The minimum clear concrete cover over reinforcement is two inches except that the minimum clear cover is three inches when the concrete is deposited on or against earth. In the structural design of slabs or beams without web reinforcement, the distance from the surface of the concrete to the centerline of the nearest reinforcing steel may be taken as 2 1/2 or 3 1/2 inches, as the case may be, to simplify the determination of the effective depth, for all bars one inch or less in diameter.

Consideration should be given to increasing the concrete cover when the surface of a slab will be exposed to high flow velocities and the water carries abrasive materials.

Temperature and Shrinkage Reinforcement

Reinforcing steel is required in both faces and in both (orthogonal) directions in all concrete slabs and walls, except that only one grid of reinforcing is required in concrete linings of trapezoidal channels. The steel serves either as principal reinforcement or as temperature and shrinkage reinforcement. The function of temperature and shrinkage reinforcement is not to eliminate cracks, it is to induce a sufficient number of small cracks so that no crack has excessive width. Well laid out temperature and shrinkage steel also serves the important auxiliary function of tying the structure together.

Where principal steel is required in only one direction, it shall ordinarily be placed nearer the concrete surface than the temperature steel. Where principal steel is required in both directions, the steel that carries the larger moment shall ordinarily be placed nearer the concrete surface. Where principal steel is required in neither direction, the temperature steel parallel to the longer dimension of the slab or wall will ordinarily be placed nearer the concrete surface.

The minimum steel area, for slabs and walls having thickness equal to or less than 32 inches, in each face and in each direction, expressed as the ratio, ρ_t, of reinforcement area, A_s, to gross concrete area, bt, are as follows:

Steel in the direction in which the distance between expansion or contraction joints does not exceed thirty feet,

ρ_t = 0.002 in the exposed face

ρ_t = 0.001 in the unexposed face.

Steel in the direction in which the distance between expansion or contraction joints exceeds thirty feet,

ρ_t = 0.003 in the exposed face

ρ_t = 0.002 in the unexposed face.

The minimum steel area for slabs and walls having thicknesses greater than 32 inches shall be computed as though the thickness were 32 inches

When expansion or contraction in a member is restrained along any line, the concept of equivalent distance between expansion or contraction joints should be used to determine the required steel ratio, ρ_t. The equivalent distance is taken as double the perpendicular distance from the line of restraint to the far edge or line of support of the member.

When the surface of a wall or slab will be exposed for a considerable period during construction, the steel provided should satisfy requirements for an exposed face.

Where a single grid of reinforcement is used, as permitted above, the steel ratio, ρ_t, shall be the sum of that listed for both faces.

Splices and development lengths for temperature and shrinkage reinforcement shall be designed for the full yield strength, f_y.

Spacing of Reinforcement

The maximum spacing of principal steel shall be twice the thickness of the slab or wall, but not more than 18 inches. The maximum spacing of temperature and shrinkage steel shall be three times the thickness of the slab or wall, but not more than 18 inches.

The clear distance between parallel bars in a layer shall not be less than the bar diameter, 1 1/3 times the maximum size of the coarse aggregate, nor 1 inch.

Where parallel reinforcement is placed in two or more layers, bars in the upper layers shall be placed directly above bars in the bottom layer. The clear distance between layers shall not be less than 1 inch.

The clear distance between bars also applies to the clear distance between a contact lap splice and adjacent splices or bars.

Minimum Tensile Reinforcement for Beams

This provision applies to flexural members other than slabs. Minimum requirements for slabs are specified under temperature and shrinkage requirements.

With sufficiently small amounts of reinforcement, failure of a section by flexure would be very sudden. This would occur when the moment strength computed as a reinforced concrete section is less than the moment strength computed as a plain concrete section.

To prevent such failures, a minimum steel requirement is specified. At any section where reinforcement is required by analysis, the steel ratio, ρ_{bm}, provided shall not be less than

$$\rho_{bm} = 200/f_y \tag{91}$$

unless the reinforcement provided at the section is at least one-third greater than that required by the analysis.

Steel in the direction in which the distance between expansion or contraction joints exceeds thirty feet,

ρ_t = 0.003 in the exposed face

ρ_t = 0.002 in the unexposed face.

The minimum steel area for slabs and walls having thicknesses greater than 32 inches shall be computed as though the thickness were 32 inches

When expansion or contraction in a member is restrained along any line, the concept of equivalent distance between expansion or contraction joints should be used to determine the required steel ratio, ρ_t. The equivalent distance is taken as double the perpendicular distance from the line of restraint to the far edge or line of support of the member.

When the surface of a wall or slab will be exposed for a considerable period during construction, the steel provided should satisfy requirements for an exposed face.

Where a single grid of reinforcement is used, as permitted above, the steel ratio, ρ_t, shall be the sum of that listed for both faces.

Splices and development lengths for temperature and shrinkage reinforcement shall be designed for the full yield strength, f_y.

<u>Spacing of Reinforcement</u>

The maximum spacing of principal steel shall be twice the thickness of the slab or wall, but not more than 18 inches. The maximum spacing of temperature and shrinkage steel shall be three times the thickness of the slab or wall, but not more than 18 inches.

The clear distance between parallel bars in a layer shall not be less than the bar diameter, 1 1/3 times the maximum size of the coarse aggregate, nor 1 inch.

Where parallel reinforcement is placed in two or more layers, bars in the upper layers shall be placed directly above bars in the bottom layer. The clear distance between layers shall not be less than 1 inch.

The clear distance between bars also applies to the clear distance between a contact lap splice and adjacent splices or bars.

Minimum Tensile Reinforcement for Beams

This provision applies to flexural members other than slabs. Minimum requirements for slabs are specified under temperature and shrinkage requirements.

With sufficiently small amounts of reinforcement, failure of a section by flexure would be very sudden. This would occur when the moment strength computed as a reinforced concrete section is less than the moment strength computed as a plain concrete section.

To prevent such failures, a minimum steel requirement is specified. At any section where reinforcement is required by analysis, the steel ratio, ρ_{bm}, provided shall not be less than

$$\rho_{bm} = 200/f_y \tag{91}$$

unless the reinforcement provided at the section is at least one-third greater than that required by the analysis.

Table 1. Flexure steel ratios.

Grade of Steel	Class of Concrete	$\bar{\rho}_b$	$0.5\ \bar{\rho}_b$	ρ_{shy}	$\rho_{shy}/\bar{\rho}_b$
60	6000	0.03773	0.01886	0.01368	0.36
	5000	0.03354	0.01677	0.01073	0.32
	4000	0.02851	0.01425	0.00795	0.28
	3000	0.02138	0.01069	0.00538	0.25
	2500	0.01782	0.00891	0.00419	0.24
50	6000	0.04858	0.02429	0.01844	0.38
	5000	0.04318	0.02159	0.01451	0.34
	4000	0.03671	0.01835	0.01080	0.29
	3000	0.02753	0.01376	0.00735	0.27
	2500	0.02294	0.01147	0.00574	0.25
40	6000	0.06551	0.03275	0.02629	0.40
	5000	0.05823	0.02911	0.02079	0.36
	4000	0.04949	0.02475	0.01556	0.31
	3000	0.03712	0.01856	0.01066	0.29
	2500	0.03093	0.01547	0.00837	0.27

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN. AREAS AND PERIMETERS OF BARS AT VARIOUS SPACINGS FOR 12 INCH WIDTH

Areas given in top figures in square inches
Perimeters given in bottom figures in inches

Spacing	Bar Size									Spacing
	#3	#4	#5	#6	#7	#8	#9	#10	#11	
2	0.66 7.07	1.18 9.42	1.84 11.78	2.65 14.14						2
2 1/4	0.59 6.28	1.05 8.38	1.64 10.47	2.36 12.57	3.21 14.66					2 1/4
2 1/2	0.53 5.65	0.94 7.54	1.47 9.42	2.12 11.31	2.89 13.20	3.77 15.08				2 1/2
2 3/4	0.48 5.14	0.86 6.85	1.34 8.57	1.93 10.28	2.62 12.00	3.43 13.71				2 3/4
3	0.44 4.71	0.79 6.28	1.23 7.85	1.77 9.42	2.40 11.00	3.14 12.57	4.00 14.18			3
3 1/4	0.41 4.35	0.73 5.80	1.13 7.25	1.63 8.70	2.22 10.15	2.90 11.60	3.69 13.09			3 1/4
3 1/2	0.38 4.04	0.67 5.39	1.05 6.73	1.51 8.08	2.06 9.42	2.69 10.77	3.43 12.15	4.34 13.68		3 1/2
3 3/4	0.35 3.77	0.63 5.03	0.98 6.28	1.41 7.54	1.92 8.80	2.51 10.05	3.20 11.34	4.05 12.77	5.00 14.18	3 3/4
4	0.33 3.53	0.59 4.71	0.92 5.89	1.33 7.07	1.80 8.25	2.36 9.42	3.00 10.63	3.80 11.97	4.69 13.29	4
4 1/4	0.31 3.33	0.55 4.44	0.87 5.54	1.25 6.65	1.70 7.76	2.22 8.87	2.82 10.01	3.57 11.27	4.41 12.51	4 1/4
4 1/2	0.29 3.14	0.52 4.19	0.82 5.24	1.18 6.28	1.60 7.33	2.09 8.38	2.67 9.45	3.37 10.64	4.17 11.81	4 1/2
4 3/4	0.28 2.98	0.50 3.97	0.78 4.96	1.12 5.95	1.50 6.94	1.98 7.94	2.53 8.95	3.20 10.08	3.95 11.19	4 3/4
5	0.26 2.83	0.47 3.77	0.74 4.71	1.06 5.65	1.44 6.60	1.88 7.54	2.40 8.51	3.04 9.58	3.75 10.63	5
5 1/4	0.25 2.69	0.45 3.59	0.70 4.49	1.01 5.39	1.37 6.28	1.80 7.18	2.29 8.10	2.89 9.12	3.57 10.13	5 1/4
5 1/2	0.24 2.57	0.43 3.43	0.67 4.28	0.96 5.14	1.31 6.00	1.71 6.85	2.18 7.73	2.76 8.71	3.41 9.66	5 1/2
5 3/4	0.23 2.46	0.41 3.28	0.64 4.10	0.92 4.92	1.25 5.74	1.64 6.56	2.09 7.40	2.64 8.33	3.26 9.25	5 3/4
6	0.22 2.36	0.39 3.14	0.61 3.93	0.88 4.71	1.20 5.50	1.57 6.28	2.00 7.09	2.53 7.98	3.12 8.86	6
6 1/2	0.20 2.18	0.36 2.90	0.57 3.62	0.82 4.35	1.11 5.07	1.45 5.80	1.85 6.54	2.34 7.37	2.88 8.18	6 1/2
7	0.19 2.02	0.34 2.69	0.53 3.37	0.76 4.04	1.03 4.71	1.35 5.39	1.71 6.08	2.17 6.84	2.68 7.59	7
7 1/2	0.18 1.89	0.31 2.51	0.49 3.14	0.71 3.77	0.96 4.40	1.26 5.03	1.60 5.67	2.03 6.38	2.50 7.09	7 1/2
8	0.17 1.77	0.29 2.36	0.46 2.95	0.66 3.53	0.90 4.12	1.18 4.71	1.50 5.32	1.90 5.99	2.34 6.65	8
8 1/2	0.16 1.66	0.28 2.22	0.43 2.77	0.62 3.33	0.85 3.88	1.11 4.44	1.41 5.00	1.79 5.63	2.21 6.25	8 1/2
9	0.15 1.57	0.26 2.09	0.41 2.62	0.59 3.14	0.80 3.67	1.05 4.19	1.33 4.73	1.69 5.32	2.08 5.91	9
9 1/2	0.14 1.49	0.25 1.98	0.39 2.48	0.56 2.98	0.76 3.47	0.99 3.97	1.26 4.48	1.60 5.04	1.97 5.60	9 1/2
10	0.13 1.41	0.24 1.88	0.37 2.36	0.53 2.83	0.72 3.30	0.94 3.77	1.20 4.25	1.52 4.79	1.88 5.32	10
10 1/2	0.13 1.35	0.22 1.80	0.35 2.24	0.50 2.69	0.69 3.14	0.90 3.59	1.14 4.05	1.45 4.56	1.79 5.06	10 1/2
11	0.12 1.29	0.21 1.71	0.33 2.14	0.48 2.57	0.66 3.00	0.86 3.43	1.09 3.87	1.38 4.35	1.70 4.83	11
11 1/2	0.12 1.23	0.20 1.64	0.32 2.05	0.46 2.46	0.63 2.87	0.82 3.28	1.04 3.70	1.32 4.16	1.63 4.62	11 1/2
12	0.11 1.18	0.20 1.57	0.31 1.96	0.44 2.36	0.60 2.75	0.79 3.14	1.00 3.54	1.27 3.99	1.56 4.43	12
13	0.10 1.09	0.18 1.45	0.28 1.81	0.41 2.18	0.56 2.54	0.72 2.90	0.92 3.27	1.17 3.68	1.44 4.09	13
14	0.09 1.01	0.17 1.35	0.26 1.68	0.38 2.02	0.52 2.36	0.67 2.69	0.86 3.04	1.08 3.42	1.34 3.80	14
15	0.09 0.94	0.16 1.26	0.25 1.57	0.35 1.89	0.48 2.20	0.63 2.51	0.80 2.84	1.01 3.19	1.25 3.54	15
16	0.08 0.88	0.15 1.18	0.23 1.47	0.33 1.77	0.45 2.06	0.59 2.36	0.75 2.66	0.95 2.99	1.17 3.32	16
17	0.08 0.83	0.14 1.11	0.22 1.39	0.31 1.66	0.42 1.94	0.55 2.22	0.71 2.50	0.89 2.82	1.10 3.13	17
18	0.07 0.79	0.13 1.05	0.20 1.31	0.29 1.57	0.40 1.83	0.52 2.09	0.67 2.36	0.84 2.66	1.04 2.95	18
	#3	#4	#5	#6	#7	#8	#9	#10	#11	

REFERENCE

U. S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN SECTION

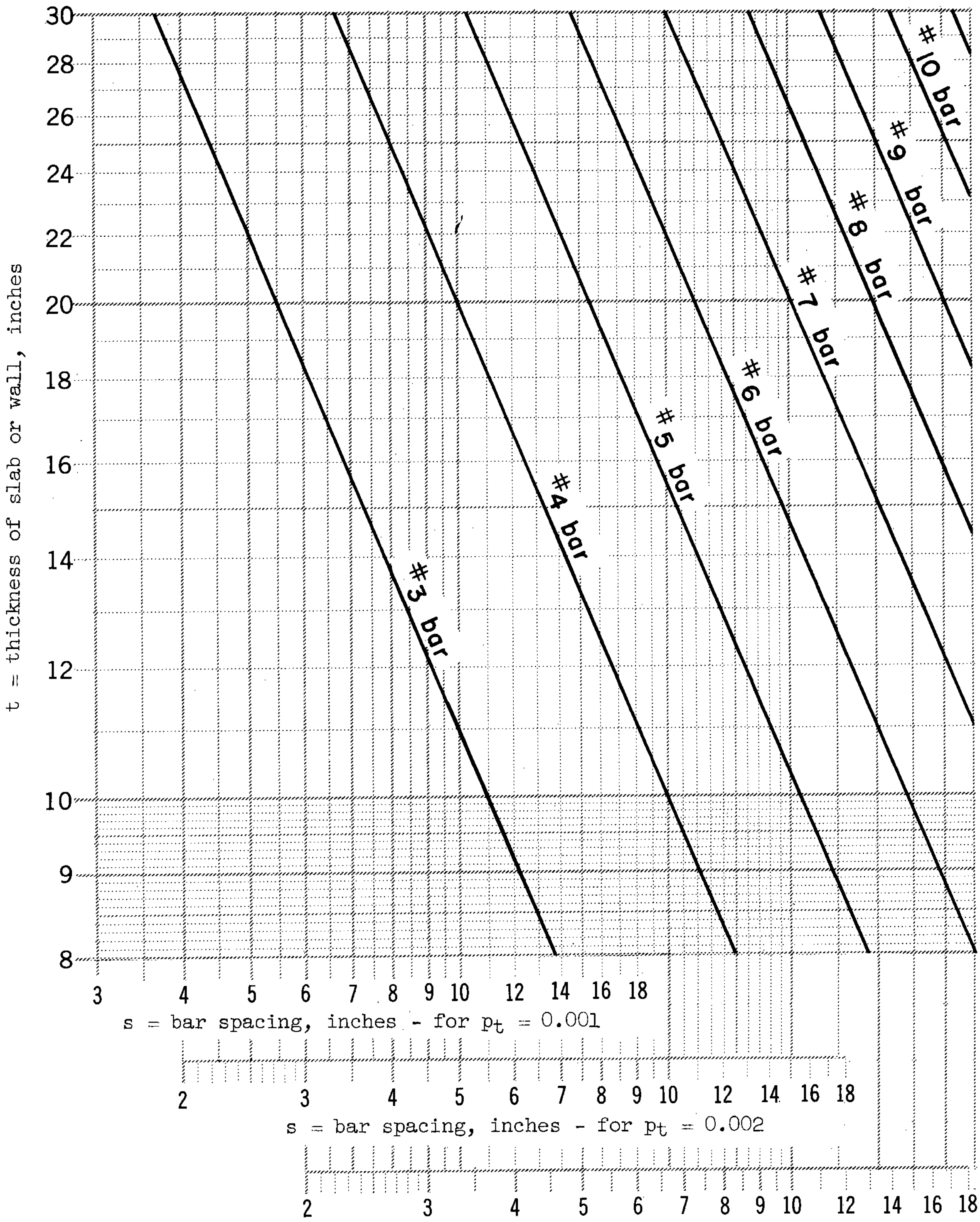
STANDARD DWG. NO.
ES – 46
SHEET 1 OF 1
DATE 12-14-50

REVISED 7-64

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN

BAR SPACINGS FOR

TEMPERATURE AND SHRINKAGE STEEL



$$s = \frac{A}{p_t t}$$

A = area of bar, sq in.

p_t = temperature and shrinkage steel ratio

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES- 47 SHEET 1 OF 1 DATE 12-8-50

REVISED 7-64

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
SINGLY REINFORCED SECTIONS, 12 INCH WIDTH

f_y = 40 ksi
f'_c = 2500 psi

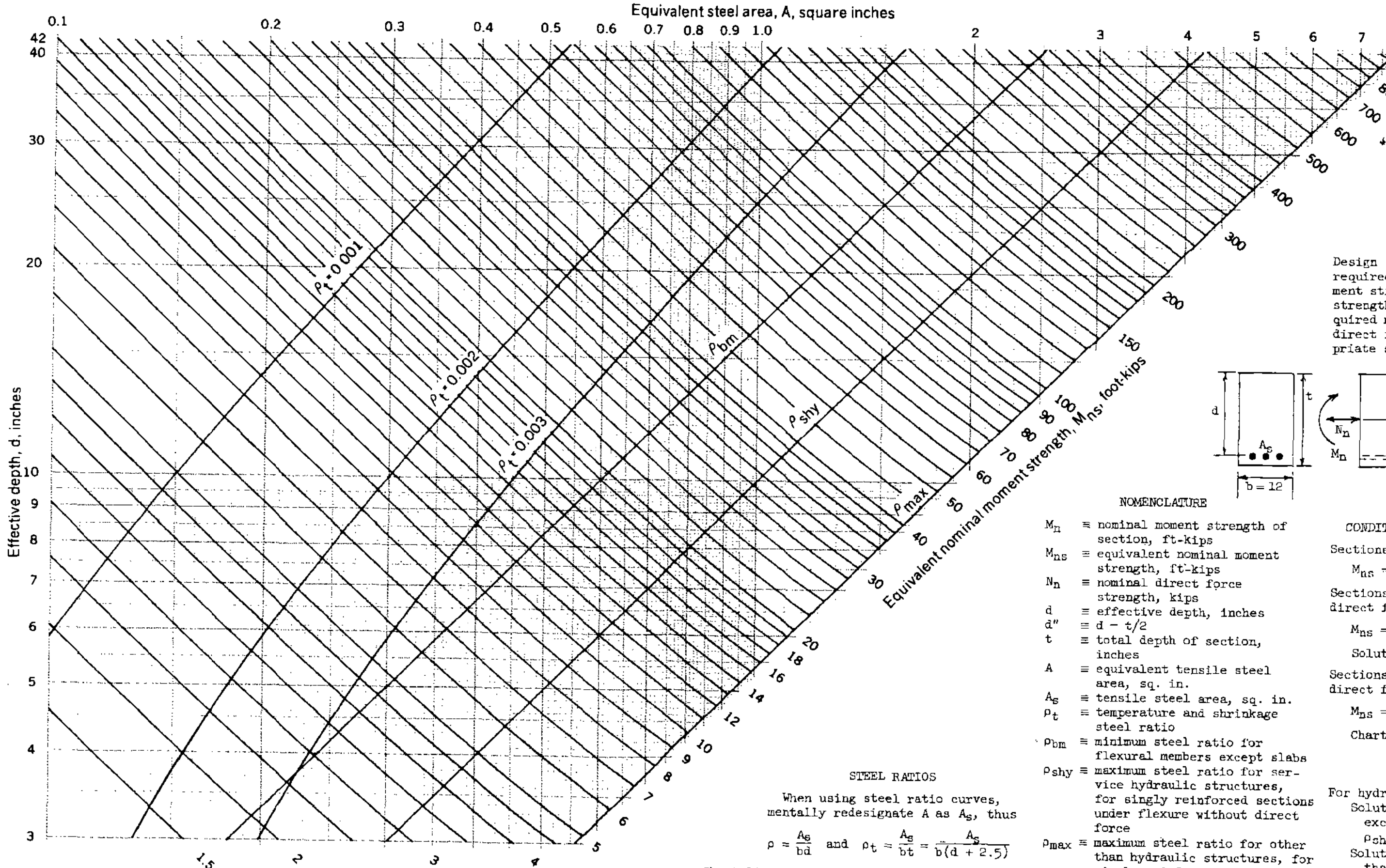
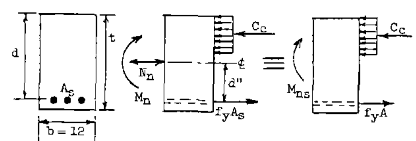


CHART CONSTANTS

f'_c = 2500 psi
E_c = 2,880,000 psi
f_y = 40 ksi
E_s = 29,000,000 psi
n = 10.1

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.



NOMENCLATURE

- M_n ≡ nominal moment strength of section, ft-kips
- M_{ns} ≡ equivalent nominal moment strength, ft-kips
- N_n ≡ nominal direct force strength, kips
- d ≡ effective depth, inches
- d" ≡ d − t/2
- t ≡ total depth of section, inches
- A ≡ equivalent tensile steel area, sq. in.
- A_s ≡ tensile steel area, sq. in.
- ρ_t ≡ temperature and shrinkage steel ratio
- ρ_{bm} ≡ minimum steel ratio for flexural members except slabs
- ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
- ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
- $\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
- ϕ ≡ strength reduction factor

STEEL RATIOS

When using steel ratio curves, mentally redesignate A as A_s, thus

$$\rho = \frac{A_s}{bd} \quad \text{and} \quad \rho_t = \frac{A_s}{bt} = \frac{A_s}{b(d + 2.5)}$$

Chart lines marked ρ_t and ρ_{bm} give minimum values of A_s for slabs and beams respectively.

For moment or moment and compressive direct force, chart lines marked ρ_{shy} and ρ_{max} give maximum values of A_s for hydraulic structures and other structures respectively.

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$$M_{ns} = M_n \quad ; \quad A_s = A$$

Sections having moment and compressive direct force:

$$M_{ns} = M_n + N_n \frac{d''}{12} \quad ; \quad A_s = A - \frac{N_n}{f_y}$$

Solution does not apply if $A_s < 0$.

Sections having moment and tensile direct force:

$$M_{ns} = M_n - N_n \frac{d''}{12} \quad ; \quad A_s = A + \frac{N_n}{f_y}$$

Chart does not apply for $M_{ns} < 0$.

LIMITATIONS

For hydraulic structures:
Solution applies for all A not exceeding A corresponding to ρ_{shy} for the given d.
Solution applies for A greater than A corresponding to ρ_{shy}, say $A_{\rho_{shy}}$, for the given d if

$$N_n \geq f_y(A - A_{\rho_{shy}})$$

For other than hydraulic structures:
Solution applies for all A not exceeding A corresponding to ρ_{max} for the given d.

REFERENCE

U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

STANDARD DWG. NO.
ES-220
SHEET 1 OF 5
DATE 12-78

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
SINGLY REINFORCED SECTIONS, 12 INCH WIDTH

$f_y = 40$ ksi

$f'_c = 3000$ psi

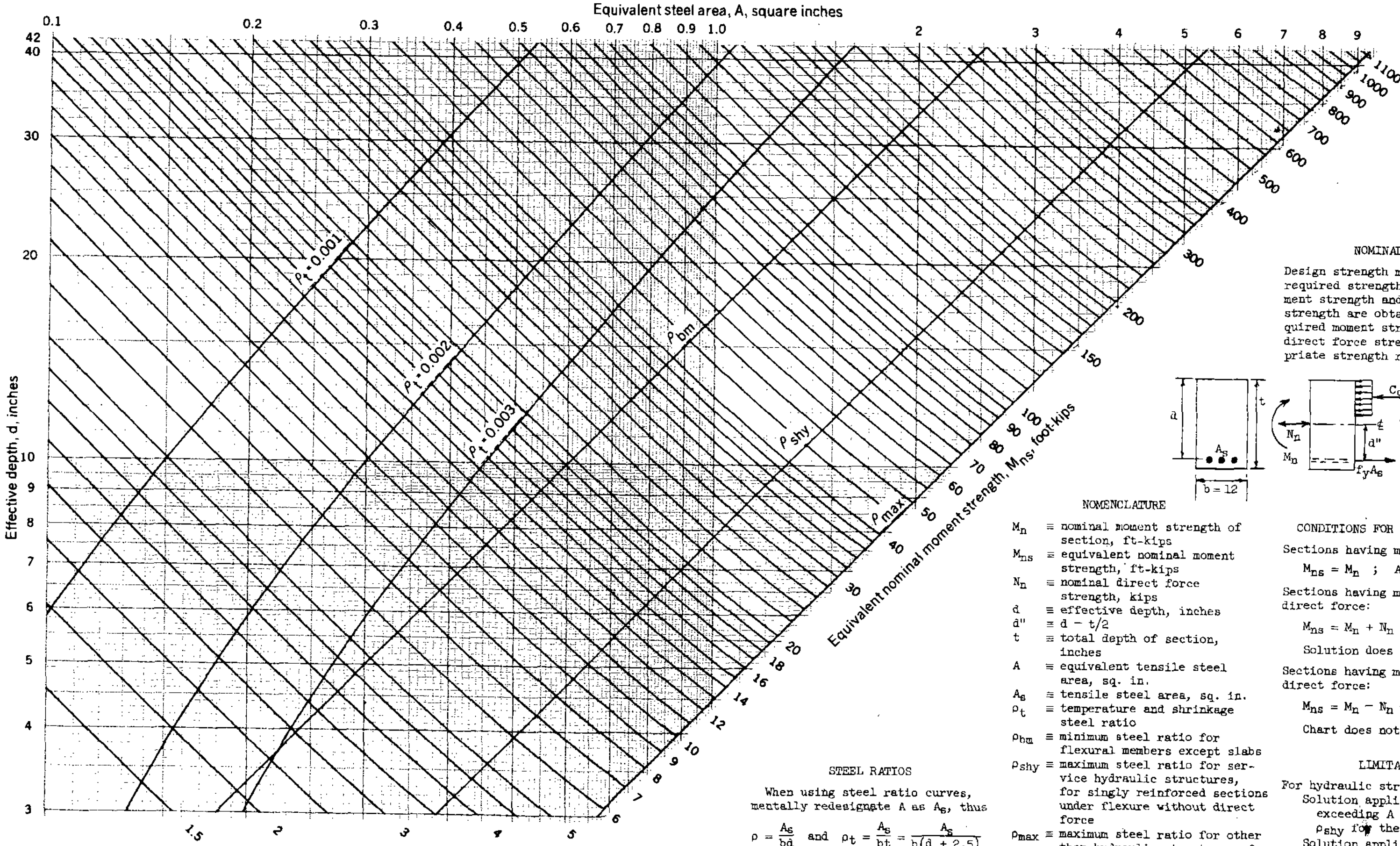


CHART CONSTANTS

f'_c = 3000 psi
E_c = 3,160,000 psi
f_y = 40 ksi
E_s = 29,000,000 psi
n = 9.2

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.

NOMENCLATURE

- M_n ≡ nominal moment strength of section, ft-kips
- M_{ns} ≡ equivalent nominal moment strength, ft-kips
- N_n ≡ nominal direct force strength, kips
- d ≡ effective depth, inches
- d" ≡ d − t/2
- t ≡ total depth of section, inches
- A ≡ equivalent tensile steel area, sq. in.
- A_s ≡ tensile steel area, sq. in.
- ρ_t ≡ temperature and shrinkage steel ratio
- ρ_{bm} ≡ minimum steel ratio for flexural members except slabs
- ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
- ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
- $\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
- ϕ ≡ strength reduction factor

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$M_{ns} = M_n$; $A_s = A$

Sections having moment and compressive direct force:

$M_{ns} = M_n + N_n \frac{d''}{12}$; $A_s = A - \frac{N_n}{f_y}$

Solution does not apply if $A_s < 0$.

Sections having moment and tensile direct force:

$M_{ns} = M_n - N_n \frac{d''}{12}$; $A_s = A + \frac{N_n}{f_y}$

Chart does not apply for $M_{ns} < 0$.

LIMITATIONS

For hydraulic structures:
Solution applies for all A not exceeding A corresponding to ρ_{shy} for the given d.
Solution applies for A greater than A corresponding to ρ_{shy}, say $A_{\rho shy}$, for the given d if

$N_n \geq f_y(A - A_{\rho shy})$

For other than hydraulic structures:
Solution applies for all A not exceeding A corresponding to ρ_{max} for the given d.

STEEL RATIOS

When using steel ratio curves, mentally redesignate A as A_s, thus

$\rho = \frac{A_s}{bd}$ and $\rho_t = \frac{A_s}{bt} = \frac{A_s}{b(d + 2.5)}$

Chart lines marked ρ_t and ρ_{bm} give minimum values of A_s for slabs and beams respectively.

For moment or moment and compressive direct force, chart lines marked ρ_{shy} and ρ_{max} give maximum values of A_s for hydraulic structures and other structures respectively.

REFERENCE

U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

STANDARD DWG. NO.
ES-220
SHEET 2 OF 5
DATE 12-78

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
SINGLY REINFORCED SECTIONS, 12 INCH WIDTH

$f_y = 40$ ksi

$f'_c = 4000$ psi

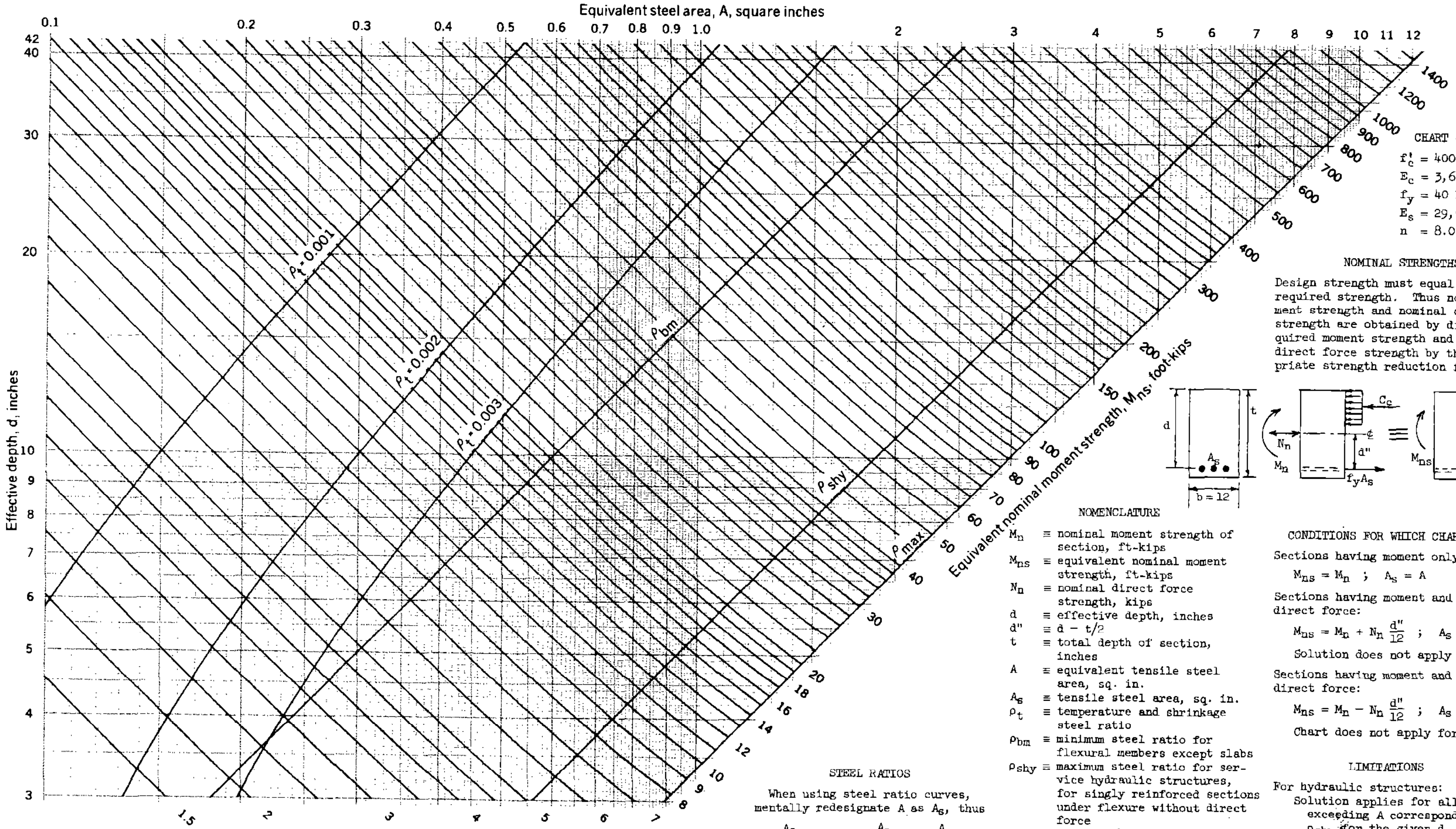


CHART CONSTANTS

$f'_c = 4000$ psi

$E_c = 3,600,000$ psi

$f_y = 40$ ksi

$E_s = 29,000,000$ psi

$n = 8.0$

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.

STEEL RATIOS

When using steel ratio curves, mentally redesignate A as A_s, thus

$$\rho = \frac{A_s}{bd} \quad \text{and} \quad \rho_t = \frac{A_s}{bt} = \frac{A_s}{b(d + 2.5)}$$

Chart lines marked ρ_t and ρ_{bm} give minimum values of A_s for slabs and beams respectively.

For moment or moment and compressive direct force, chart lines marked ρ_{shy} and ρ_{max} give maximum values of A_s for hydraulic structures and other structures respectively.

NOMENCLATURE

- M_n ≡ nominal moment strength of section, ft-kips
- M_{ns} ≡ equivalent nominal moment strength, ft-kips
- N_n ≡ nominal direct force strength, kips
- d ≡ effective depth, inches
- d'' ≡ $d - t/2$
- t ≡ total depth of section, inches
- A ≡ equivalent tensile steel area, sq. in.
- A_s ≡ tensile steel area, sq. in.
- ρ_t ≡ temperature and shrinkage steel ratio
- ρ_{bm} ≡ minimum steel ratio for flexural members except slabs
- ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
- ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
- $\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
- ϕ ≡ strength reduction factor

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$M_{ns} = M_n$; $A_s = A$

Sections having moment and compressive direct force:

$M_{ns} = M_n + N_n \frac{d''}{12}$; $A_s = A - \frac{N_n}{f_y}$

Solution does not apply if $A_s < 0$.

Sections having moment and tensile direct force:

$M_{ns} = M_n - N_n \frac{d''}{12}$; $A_s = A + \frac{N_n}{f_y}$

Chart does not apply for $M_{ns} < 0$.

LIMITATIONS

For hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{shy} for the given d.

Solution applies for A greater than A corresponding to ρ_{shy}, say $A_{\rho_{shy}}$, for the given d if

$$N_n \geq f_y(A - A_{\rho_{shy}})$$

For other than hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{max} for the given d.

REFERENCE

U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

STANDARD DWG. NO.
ES-220
SHEET 3 OF 5
DATE 12-78

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
SINGLY REINFORCED SECTIONS, 12 INCH WIDTH

f_y = 40 ksi

f'_c = 5000 psi

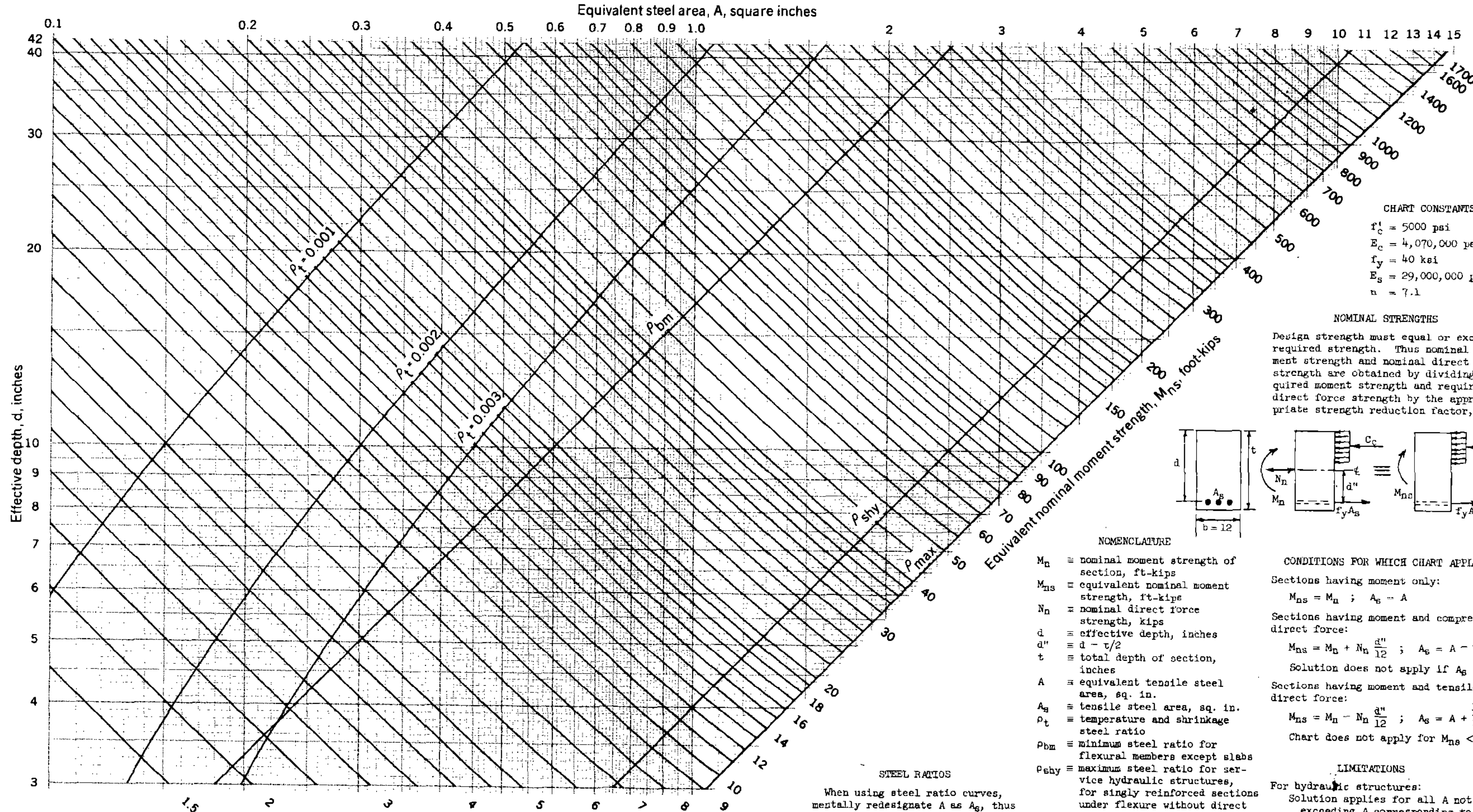


CHART CONSTANTS

f'_c = 5000 psi
E_c = 4,070,000 psi
f_y = 40 ksi
E_s = 29,000,000 psi
n = 7.1

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.

STEEL RATIOS

When using steel ratio curves, mentally redesignate A as A_s, thus

$$\rho = \frac{A_s}{bd} \quad \text{and} \quad \rho_t = \frac{A_s}{bt} = \frac{A_s}{b(d + 2.5)}$$

Chart lines marked ρ_t and ρ_{bm} give minimum values of A_s for slabs and beams respectively.

For moment or moment and compressive direct force, chart lines marked ρ_{shy} and ρ_{max} give maximum values of A_s for hydraulic structures and other structures respectively.

NOMENCLATURE

- M_n ≡ nominal moment strength of section, ft-kips
- M_{ns} ≡ equivalent nominal moment strength, ft-kips
- N_n ≡ nominal direct force strength, kips
- d ≡ effective depth, inches
- d" ≡ d − t/2
- t ≡ total depth of section, inches
- A ≡ equivalent tensile steel area, sq. in.
- A_s ≡ tensile steel area, sq. in.
- ρ_t ≡ temperature and shrinkage steel ratio
- ρ_{bm} ≡ minimum steel ratio for flexural members except slabs
- ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
- ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
- $\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
- ϕ ≡ strength reduction factor

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$M_{ns} = M_n$; $A_s = A$

Sections having moment and compressive direct force:

$M_{ns} = M_n + N_n \frac{d''}{12}$; $A_s = A - \frac{N_n}{f_y}$

Solution does not apply if $A_s < 0$

Sections having moment and tensile direct force:

$M_{ns} = M_n - N_n \frac{d''}{12}$; $A_s = A + \frac{N_n}{f_y}$

Chart does not apply for $M_{ns} < 0$.

LIMITATIONS

For hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{shy} for the given d.

Solution applies for A greater than A corresponding to ρ_{shy}, say $A_{\rho_{shy}}$, for the given d if

$N_n \geq f_y(A - A_{\rho_{shy}})$

For other than hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{max} for the given d.

REFERENCE

U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

STANDARD DWG. NO.
ES-220
SHEET 4 OF 5
DATE 12-78

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
SINGLY REINFORCED SECTIONS, 12 INCH WIDTH

f_y = 40 ksi

f'_c = 6000 psi

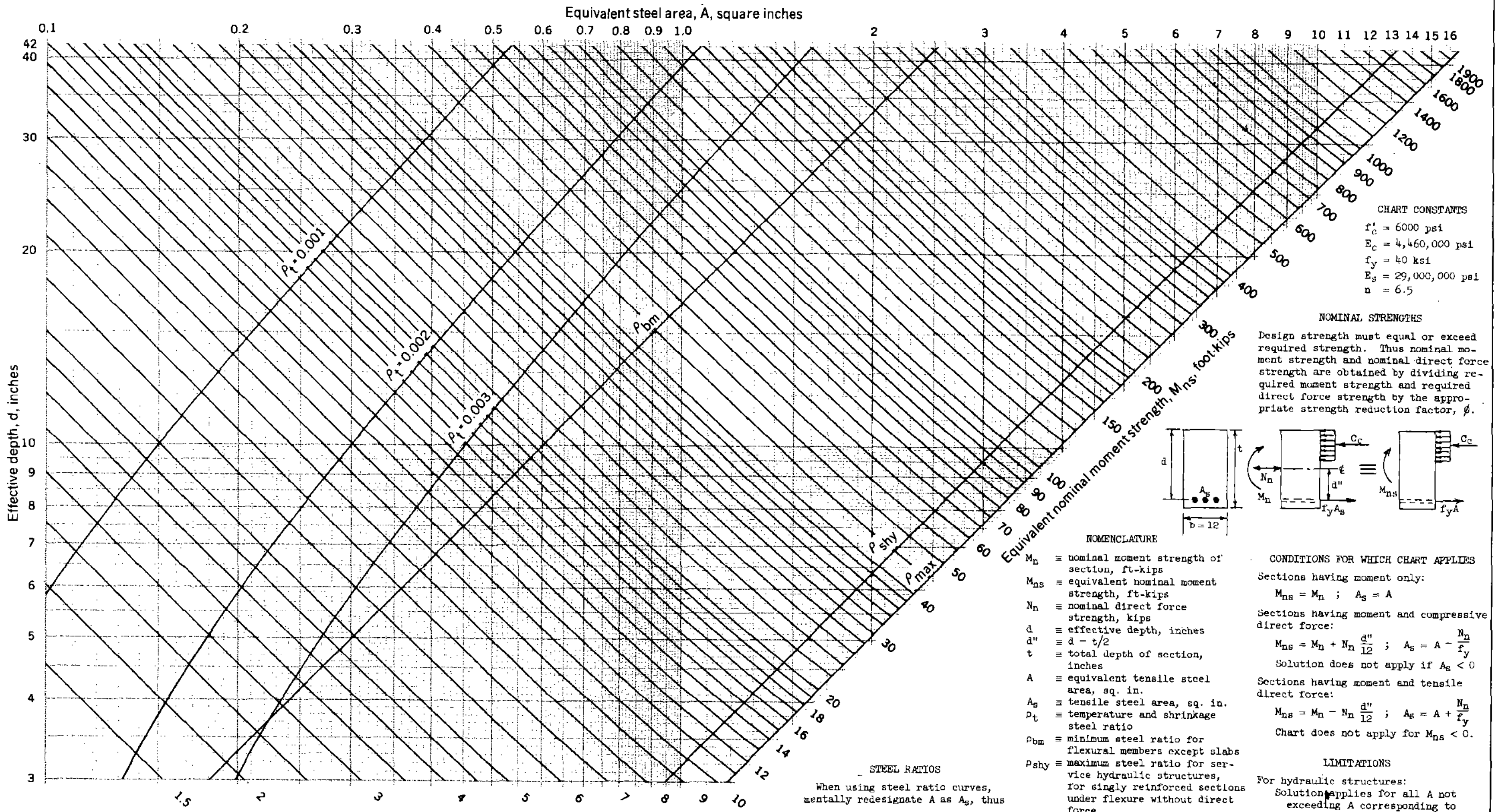


CHART CONSTANTS

f'_c = 6000 psi
E_c = 4,460,000 psi
f_y = 40 ksi
E_s = 29,000,000 psi
n = 6.5

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.

STEEL RATIOS

When using steel ratio curves, mentally redesignate A as A_s, thus

$$\rho = \frac{A_s}{bd} \quad \text{and} \quad \rho_t = \frac{A_s}{bt} = \frac{A_s}{b(d + 2.5)}$$

Chart lines marked ρ_t and ρ_{bm} give minimum values of A_s for slabs and beams respectively.

For moment or moment and compressive direct force, chart lines marked ρ_{shy} and ρ_{max} give maximum values of A_s for hydraulic structures and other structures respectively.

NOMENCLATURE

- M_n ≡ nominal moment strength of section, ft-kips
- M_{ns} ≡ equivalent nominal moment strength, ft-kips
- N_n ≡ nominal direct force strength, kips
- d ≡ effective depth, inches
- d" ≡ d − t/2
- t ≡ total depth of section, inches
- A ≡ equivalent tensile steel area, sq. in.
- A_s ≡ tensile steel area, sq. in.
- ρ_t ≡ temperature and shrinkage steel ratio
- ρ_{bm} ≡ minimum steel ratio for flexural members except slabs
- ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
- ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
- $\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
- ϕ ≡ strength reduction factor

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$M_{ns} = M_n$; $A_s = A$

Sections having moment and compressive direct force:

$M_{ns} = M_n + N_n \frac{d''}{12}$; $A_s = A - \frac{N_n}{f_y}$

Solution does not apply if $A_s < 0$

Sections having moment and tensile direct force:

$M_{ns} = M_n - N_n \frac{d''}{12}$; $A_s = A + \frac{N_n}{f_y}$

Chart does not apply for $M_{ns} < 0$.

LIMITATIONS

For hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{shy} for the given d.

Solution applies for A greater than A corresponding to ρ_{shy}, say $A_{\rho_{shy}}$, for the given d if

$N_n \geq f_y(A - A_{\rho_{shy}})$

For other than hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{max} for the given d.

REFERENCE

U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

STANDARD DWG. NO.
ES-220
SHEET 5 OF 5
DATE 12-78

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
SINGLY REINFORCED SECTIONS, 12 INCH WIDTH

f_y = 50 ksi
f'_c = 2500 psi

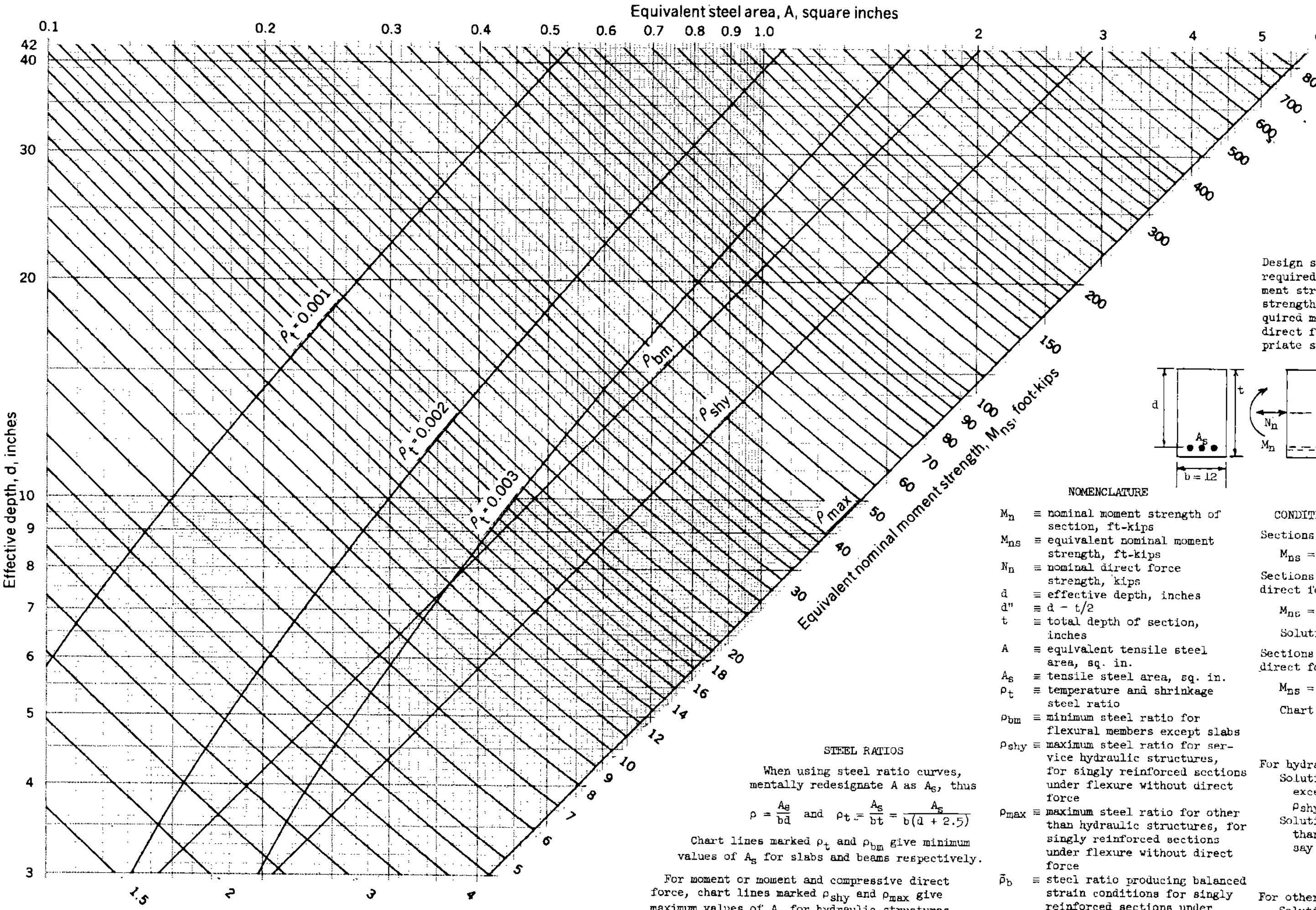
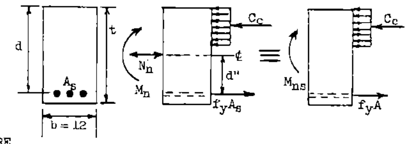


CHART CONSTANTS

f'_c = 2500 psi
E_c = 2,880,000 psi
f_y = 50 ksi
E_s = 29,000,000 psi
n = 10.1

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.



STEEL RATIOS

When using steel ratio curves, mentally redesignate A as A_s, thus

$$\rho = \frac{A_s}{bd} \quad \text{and} \quad \rho_t = \frac{A_s}{bt} = \frac{A_s}{b(d + 2.5)}$$

Chart lines marked ρ_t and ρ_{bm} give minimum values of A_s for slabs and beams respectively.

For moment or moment and compressive direct force, chart lines marked ρ_{shy} and ρ_{max} give maximum values of A_s for hydraulic structures and other structures respectively.

NOMENCLATURE

- M_n ≡ nominal moment strength of section, ft-kips
- M_{ns} ≡ equivalent nominal moment strength, ft-kips
- N_n ≡ nominal direct force strength, kips
- d ≡ effective depth, inches
- d" ≡ d − t/2
- t ≡ total depth of section, inches
- A ≡ equivalent tensile steel area, sq. in.
- A_s ≡ tensile steel area, sq. in.
- ρ_t ≡ temperature and shrinkage steel ratio
- ρ_{bm} ≡ minimum steel ratio for flexural members except slabs
- ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
- ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
- $\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
- ϕ ≡ strength reduction factor

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$M_{ns} = M_n$; $A_s = A$

Sections having moment and compressive direct force:

$M_{ns} = M_n + N_n \frac{d''}{12}$; $A_s = A - \frac{N_n}{f_y}$

Solution does not apply if $A_s < 0$

Sections having moment and tensile direct force:

$M_{ns} = M_n - N_n \frac{d''}{12}$; $A_s = A + \frac{N_n}{f_y}$

Chart does not apply for $M_{ns} < 0$.

LIMITATIONS

For hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{shy} for the given d.

Solution applies for A greater than A corresponding to ρ_{shy}, say $A_{\rho_{shy}}$, for the given d if

$N_n \geq f_y(A - A_{\rho_{shy}})$

For other than hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{max} for the given d.

REFERENCE

U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

STANDARD DWG. NO.
ES-221
SHEET 1 OF 5
DATE 12-78

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
SINGLY REINFORCED SECTIONS, 12 INCH WIDTH

f_y = 50 ksi

f'_c = 3000 psi

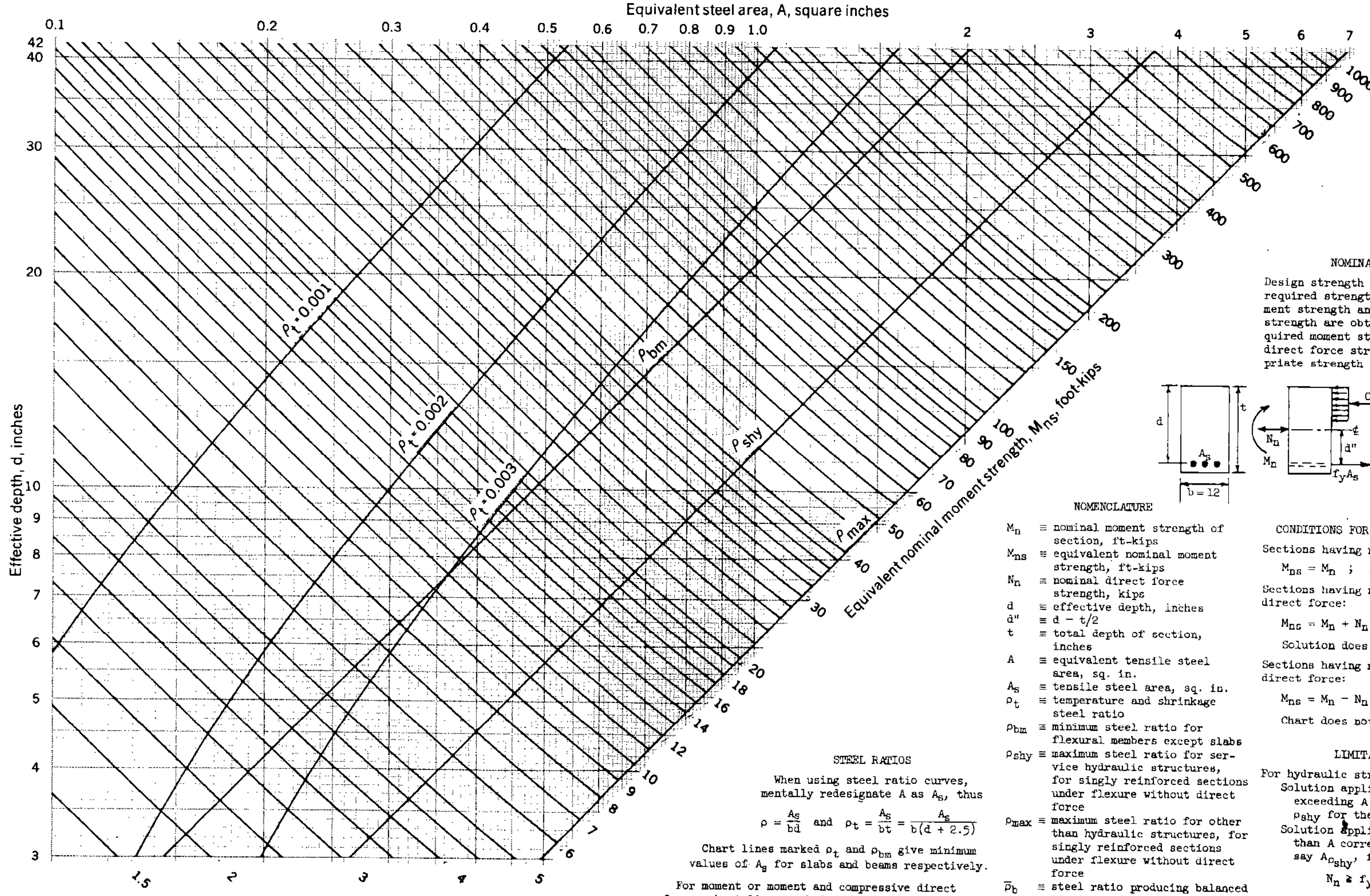


CHART CONSTANTS

f'_c = 3000 psi
E_c = 3,160,000 psi
f_y = 50 ksi
E_s = 29,000,000 psi
n = 9.2

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.

STEEL RATIOS

When using steel ratio curves, mentally redesignate A as A_s, thus

$$\rho = \frac{A_s}{bd} \quad \text{and} \quad \rho_t = \frac{A_s}{bt} = \frac{A_s}{b(d + 2.5)}$$

Chart lines marked ρ_t and ρ_{bm} give minimum values of A_s for slabs and beams respectively.

For moment or moment and compressive direct force, chart lines marked ρ_{shy} and ρ_{max} give maximum values of A_s for hydraulic structures and other structures respectively.

NOMENCLATURE

M_n ≡ nominal moment strength of section, ft-kips
M_{ns} ≡ equivalent nominal moment strength, ft-kips
N_n ≡ nominal direct force strength, kips
d ≡ effective depth, inches
d" ≡ d − t/2
t ≡ total depth of section, inches
A ≡ equivalent tensile steel area, sq. in.
A_s ≡ tensile steel area, sq. in.
ρ_t ≡ temperature and shrinkage steel ratio
ρ_{bm} ≡ minimum steel ratio for flexural members except slabs
ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
$\overline{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
ϕ ≡ strength reduction factor

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$M_{ns} = M_n$; $A_s = A$

Sections having moment and compressive direct force:

$M_{ns} = M_n + N_n \frac{d"}{12}$; $A_s = A - \frac{N_n}{f_y}$

Solution does not apply if $A_s < 0$

Sections having moment and tensile direct force:

$M_{ns} = M_n - N_n \frac{d"}{12}$; $A_s = A + \frac{N_n}{f_y}$

Chart does not apply for $M_{ns} < 0$.

LIMITATIONS

For hydraulic structures:
Solution applies for all A not exceeding A corresponding to ρ_{shy} for the given d.
Solution applies for A greater than A corresponding to ρ_{shy}, say $A_{\rho_{shy}}$, for the given d if

$N_n \geq f_y(A - A_{\rho_{shy}})$

For other than hydraulic structures:
Solution applies for all A not exceeding A corresponding to ρ_{max} for the given d.

REFERENCE

U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

STANDARD DWG. NO.
ES-221
SHEET 2 OF 5
DATE 12-78

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
SINGLY REINFORCED SECTIONS, 12 INCH WIDTH

f_y = 50 ksi

f'_c = 4000 psi

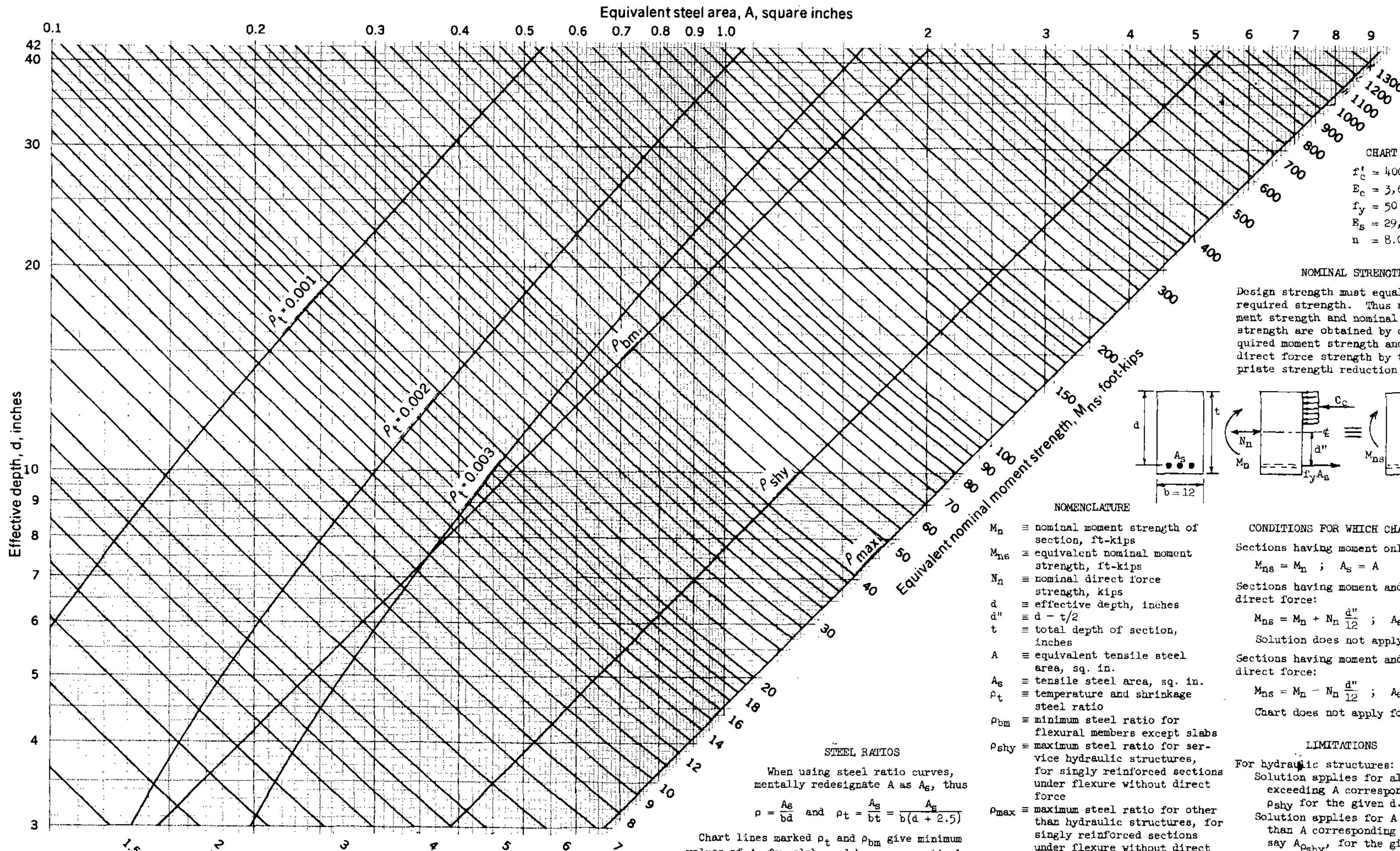


CHART CONSTANTS

f'_c = 4000 psi
E_c = 3,600,000 psi
f_y = 50 ksi
E_s = 29,000,000 psi
n = 8.0

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.

STEEL RATIOS

When using steel ratio curves, mentally redesignate A as A_s, thus

$$\rho = \frac{A_s}{bd} \quad \text{and} \quad \rho_t = \frac{A_s}{bt} = \frac{A_s}{b(d + 2.5)}$$

Chart lines marked ρ_t and ρ_{bm} give minimum values of A_s for slabs and beams respectively.

For moment or moment and compressive direct force, chart lines marked ρ_{shy} and ρ_{max} give maximum values of A_s for hydraulic structures and other structures respectively.

NOMENCLATURE

- M_n ≡ nominal moment strength of section, ft-kips
- M_{ns} ≡ equivalent nominal moment strength, ft-kips
- N_n ≡ nominal direct force strength, kips
- d ≡ effective depth, inches
- d" ≡ d − t/2
- t ≡ total depth of section, inches
- A ≡ equivalent tensile steel area, sq. in.
- A_s ≡ tensile steel area, sq. in.
- ρ_t ≡ temperature and shrinkage steel ratio
- ρ_{bm} ≡ minimum steel ratio for flexural members except slabs
- ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
- ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
- $\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
- ϕ ≡ strength reduction factor

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$M_{ns} = M_n$; $A_s = A$

Sections having moment and compressive direct force:

$M_{ns} = M_n + N_n \frac{d''}{12}$; $A_s = A - \frac{N_n}{f_y}$

Solution does not apply if $A_s < 0$

Sections having moment and tensile direct force:

$M_{ns} = M_n - N_n \frac{d''}{12}$; $A_s = A + \frac{N_n}{f_y}$

Chart does not apply for $M_{ns} < 0$.

LIMITATIONS

For hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{shy} for the given d.

Solution applies for A greater than A corresponding to ρ_{shy}, say $A_{\rho shy}$, for the given d if

$N_n \geq f_y(A - A_{\rho shy})$

For other than hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{max} for the given d.

REFERENCE

U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

STANDARD DWG. NO.
ES-221
SHEET 3 OF 5
DATE 12-78

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
SINGLY REINFORCED SECTIONS, 12 INCH WIDTH

$f_y = 50$ ksi

$f'_c = 5000$ psi

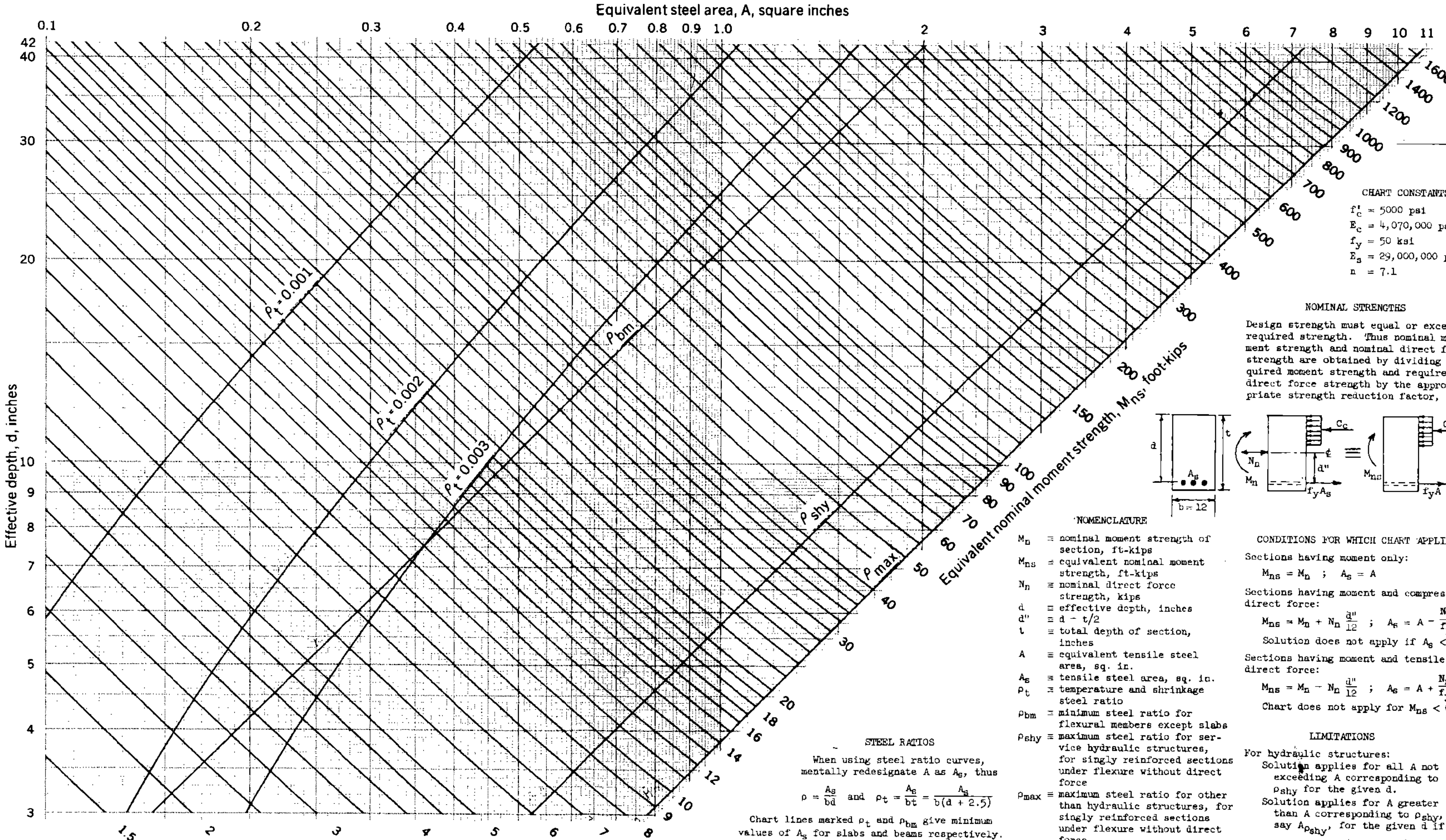


CHART CONSTANTS

$f'_c = 5000$ psi
$E_c = 4{,}070{,}000$ psi
$f_y = 50$ ksi
$E_s = 29{,}000{,}000$ psi
$n = 7.1$

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.

NOMENCLATURE

- M_n ≡ nominal moment strength of section, ft-kips
- M_{ns} ≡ equivalent nominal moment strength, ft-kips
- N_n ≡ nominal direct force strength, kips
- d ≡ effective depth, inches
- d'' ≡ $d - t/2$
- t ≡ total depth of section, inches
- A ≡ equivalent tensile steel area, sq. in.
- A_s ≡ tensile steel area, sq. in.
- ρ_t ≡ temperature and shrinkage steel ratio
- ρ_{bm} ≡ minimum steel ratio for flexural members except slabs
- ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
- ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
- $\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
- ϕ ≡ strength reduction factor

STEEL RATIOS

When using steel ratio curves, mentally redesignate A as A_s, thus

$$\rho = \frac{A_s}{bd} \quad \text{and} \quad \rho_t = \frac{A_s}{bt} = \frac{A_s}{b(d + 2.5)}$$

Chart lines marked ρ_t and ρ_{bm} give minimum values of A_s for slabs and beams respectively.

For moment or moment and compressive direct force, chart lines marked ρ_{shy} and ρ_{max} give maximum values of A_s for hydraulic structures and other structures respectively.

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$M_{ns} = M_n$; $A_s = A$

Sections having moment and compressive direct force:

$M_{ns} = M_n + N_n \frac{d''}{12}$; $A_s = A - \frac{N_n}{f_y}$

Solution does not apply if $A_s < 0$.

Sections having moment and tensile direct force:

$M_{ns} = M_n - N_n \frac{d''}{12}$; $A_s = A + \frac{N_n}{f_y}$

Chart does not apply for $M_{ns} < 0$.

LIMITATIONS

For hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{shy} for the given d.

Solution applies for A greater than A corresponding to ρ_{shy}, say $A_{\rho_{shy}}$, for the given d if

$N_n \geq f_y(A - A_{\rho_{shy}})$

For other than hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{max} for the given d.

REFERENCE

U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

STANDARD DWG. NO.
ES-221
SHEET 4 OF 5
DATE 12-78

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
SINGLY REINFORCED SECTIONS, 12 INCH WIDTH

$f_y = 50$ ksi

$f'_c = 6000$ psi

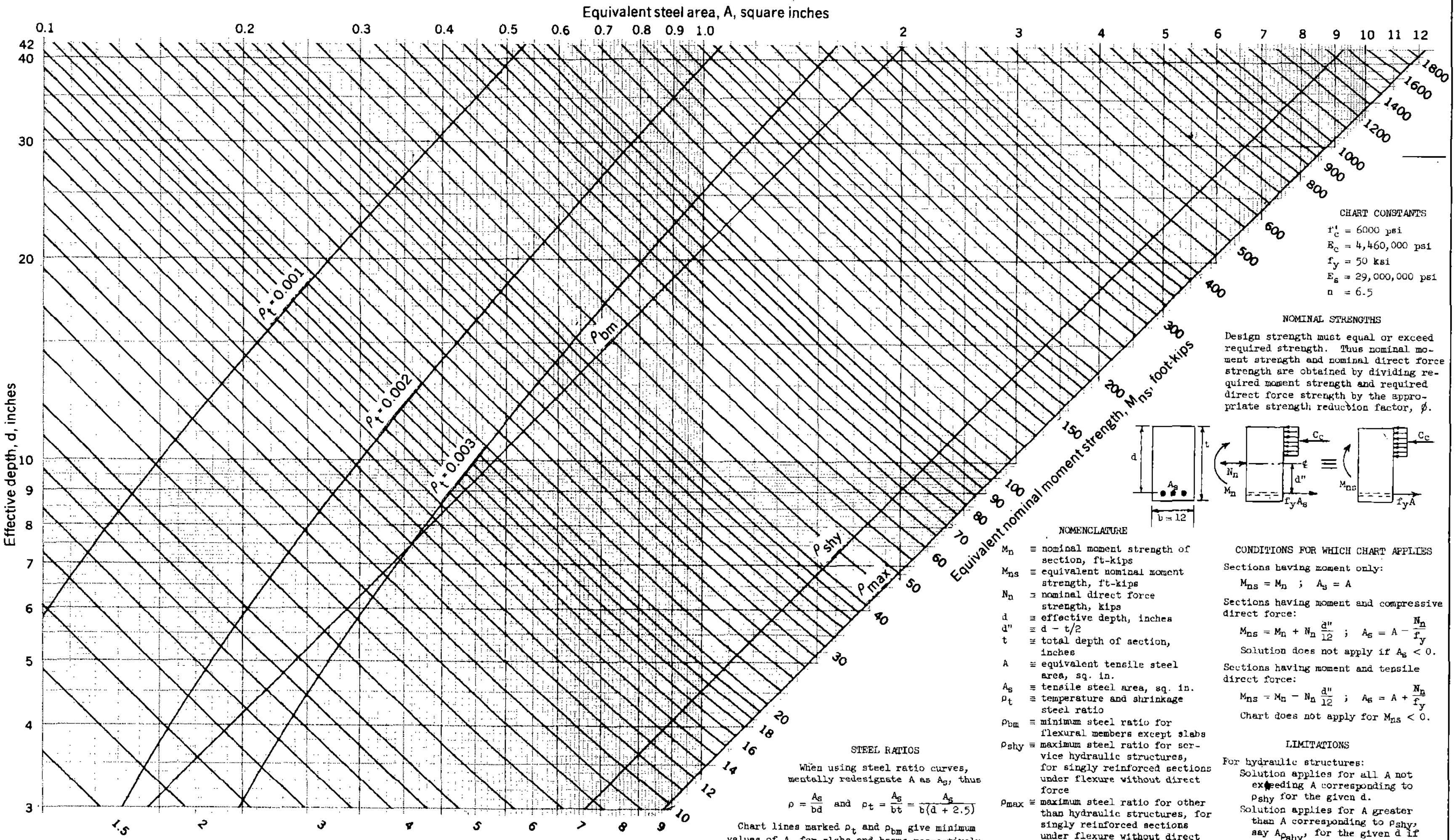


CHART CONSTANTS

$f'_c = 6000$ psi

$E_c = 4,460,000$ psi

$f_y = 50$ ksi

$E_s = 29,000,000$ psi

$n = 6.5$

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.

NOMENCLATURE

M_n ≡ nominal moment strength of section, ft-kips

M_{ns} ≡ equivalent nominal moment strength, ft-kips

N_n ≡ nominal direct force strength, kips

d ≡ effective depth, inches

d'' ≡ $d - t/2$

t ≡ total depth of section, inches

A ≡ equivalent tensile steel area, sq. in.

A_s ≡ tensile steel area, sq. in.

ρ_t ≡ temperature and shrinkage steel ratio

ρ_{bm} ≡ minimum steel ratio for flexural members except slabs

ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force

ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force

$\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force

ϕ ≡ strength reduction factor

STEEL RATIOS

When using steel ratio curves, mentally redesignate A as A_s, thus

$$\rho = \frac{A_s}{bd} \quad \text{and} \quad \rho_t = \frac{A_s}{bt} = \frac{A_s}{b(d + 2.5)}$$

Chart lines marked ρ_t and ρ_{bm} give minimum values of A_s for slabs and beams respectively.

For moment or moment and compressive direct force, chart lines marked ρ_{shy} and ρ_{max} give maximum values of A_s for hydraulic structures and other structures respectively.

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$$M_{ns} = M_n \quad ; \quad A_s = A$$

Sections having moment and compressive direct force:

$$M_{ns} = M_n + N_n \frac{d''}{12} \quad ; \quad A_s = A - \frac{N_n}{f_y}$$

Solution does not apply if $A_s < 0$.

Sections having moment and tensile direct force:

$$M_{ns} = M_n - N_n \frac{d''}{12} \quad ; \quad A_s = A + \frac{N_n}{f_y}$$

Chart does not apply for $M_{ns} < 0$.

LIMITATIONS

For hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{shy} for the given d.

Solution applies for A greater than A corresponding to ρ_{shy}, say $A_{\rho_{shy}}$, for the given d if

$$N_n \geq f_y(A - A_{\rho_{shy}})$$

For other than hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{max} for the given d.

REFERENCE

U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

STANDARD DWG. NO.
ES-221
SHEET 5 OF 5
DATE 12-78

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
SINGLY REINFORCED SECTIONS, 12 INCH WIDTH

f_y = 60 ksi

f'_c = 2500 psi

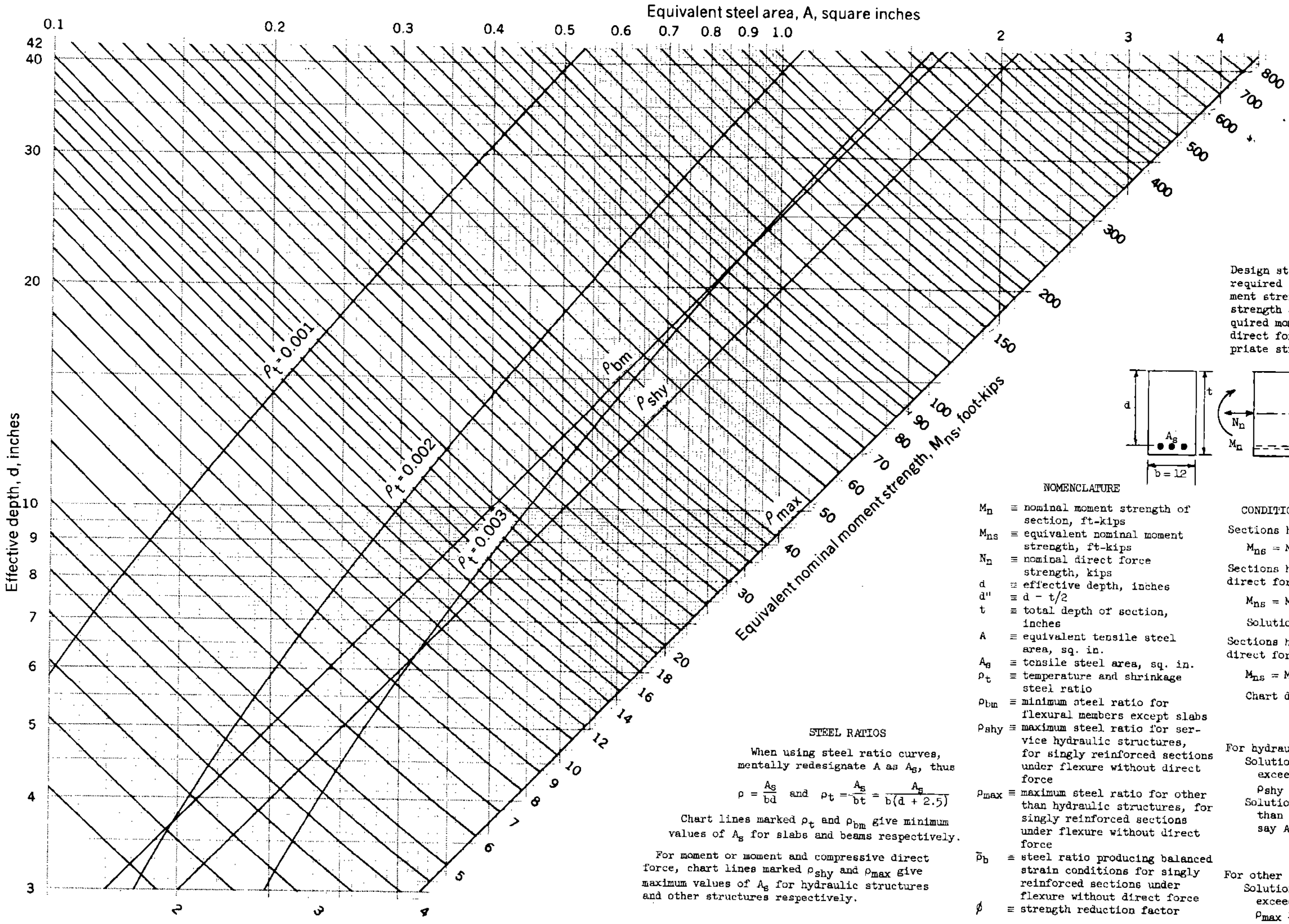


CHART CONSTANTS

f'_c = 2500 psi

E_c = 2,880,000 psi

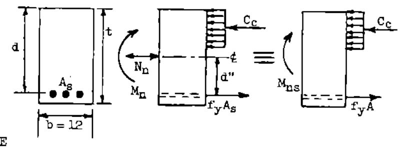
f_y = 60 ksi

E_s = 29,000,000 psi

n = 10.1

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.



NOMENCLATURE

M_n ≡ nominal moment strength of section, ft-kips

M_{ns} ≡ equivalent nominal moment strength, ft-kips

N_n ≡ nominal direct force strength, kips

d ≡ effective depth, inches

d" ≡ d − t/2

t ≡ total depth of section, inches

A ≡ equivalent tensile steel area, sq. in.

A_s ≡ tensile steel area, sq. in.

ρ_t ≡ temperature and shrinkage steel ratio

ρ_{bm} ≡ minimum steel ratio for flexural members except slabs

ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force

ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force

$\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force

ϕ ≡ strength reduction factor

STEEL RATIOS

When using steel ratio curves, mentally redesignate A as A_s, thus

$$\rho = \frac{A_s}{bd} \quad \text{and} \quad \rho_t = \frac{A_s}{bt} = \frac{A_s}{b(d + 2.5)}$$

Chart lines marked ρ_t and ρ_{bm} give minimum values of A_s for slabs and beams respectively.

For moment or moment and compressive direct force, chart lines marked ρ_{shy} and ρ_{max} give maximum values of A_s for hydraulic structures and other structures respectively.

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$$M_{ns} = M_n \quad ; \quad A_s = A$$

Sections having moment and compressive direct force:

$$M_{ns} = M_n + N_n \frac{d''}{12} \quad ; \quad A_s = A - \frac{N_n}{f_y}$$

Solution does not apply if $A_s < 0$.

Sections having moment and tensile direct force:

$$M_{ns} = M_n - N_n \frac{d''}{12} \quad ; \quad A_s = A + \frac{N_n}{f_y}$$

Chart does not apply for $M_{ns} < 0$.

LIMITATIONS

For hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{shy} for the given d.

Solution applies for A greater than A corresponding to ρ_{shy}, say $A_{\rho_{shy}}$, for the given d if

$$N_n \geq f_y(A - A_{\rho_{shy}})$$

For other than hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{max} for the given d.

REFERENCE

U.S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN UNIT	STANDARD DWG. NO. ES-222 SHEET 1 OF 5 DATE 12-78

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
SINGLY REINFORCED SECTIONS, 12 INCH WIDTH

f_y = 60 ksi
f'_c = 3000 psi

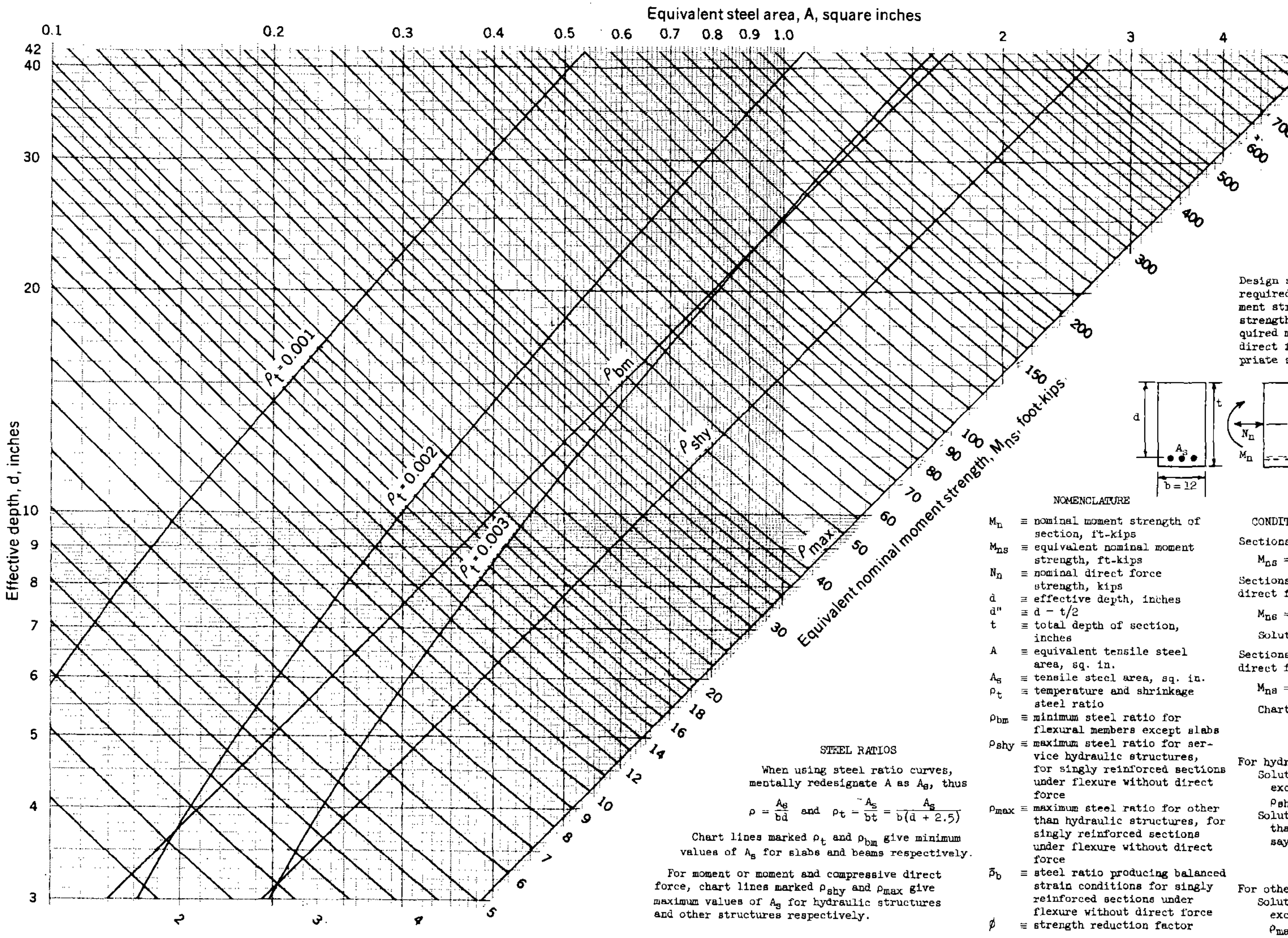
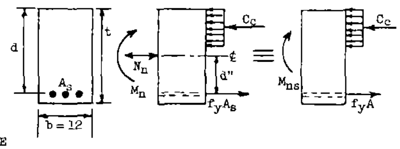


CHART CONSTANTS

f'_c = 3000 psi
E_c = 3,160,000 psi
f_y = 60 ksi
E_s = 29,000,000 psi
n = 9.2

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.



NOMENCLATURE

- M_n ≡ nominal moment strength of section, ft-kips
- M_{ns} ≡ equivalent nominal moment strength, ft-kips
- N_n ≡ nominal direct force strength, kips
- d ≡ effective depth, inches
- d" ≡ d − t/2
- t ≡ total depth of section, inches
- A ≡ equivalent tensile steel area, sq. in.
- A_s ≡ tensile steel area, sq. in.
- ρ_t ≡ temperature and shrinkage steel ratio
- ρ_{bm} ≡ minimum steel ratio for flexural members except slabs
- ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
- ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
- $\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
- ϕ ≡ strength reduction factor

STEEL RATIOS

When using steel ratio curves, mentally redesignate A as A_s, thus

$$\rho = \frac{A_s}{bd} \quad \text{and} \quad \rho_t = \frac{A_s}{bt} = \frac{A_s}{b(d + 2.5)}$$

Chart lines marked ρ_t and ρ_{bm} give minimum values of A_s for slabs and beams respectively.

For moment or moment and compressive direct force, chart lines marked ρ_{shy} and ρ_{max} give maximum values of A_s for hydraulic structures and other structures respectively.

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$M_{ns} = M_n$; $A_s = A$

Sections having moment and compressive direct force:

$M_{ns} = M_n + N_n \frac{d"}{12}$; $A_s = A - \frac{N_n}{f_y}$

Solution does not apply if $A_s < 0$.

Sections having moment and tensile direct force:

$M_{ns} = M_n - N_n \frac{d"}{12}$; $A_s = A + \frac{N_n}{f_y}$

Chart does not apply for $M_{ns} < 0$.

LIMITATIONS

For hydraulic structures:
Solution applies for all A not exceeding A corresponding to ρ_{shy} for the given d.
Solution applies for A greater than A corresponding to ρ_{shy}, say $A_{\rho_{shy}}$, for the given d if
$N_n \geq f_y(A - A_{\rho_{shy}})$

For other than hydraulic structures:
Solution applies for all A not exceeding A corresponding to ρ_{max} for the given d.

REFERENCE

U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

STANDARD DWG. NO.
ES-222
SHEET 2 OF 5
DATE 12-78

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
SINGLY REINFORCED SECTIONS, 12 INCH WIDTH

$f_y = 60$ ksi

$f'_c = 4000$ psi

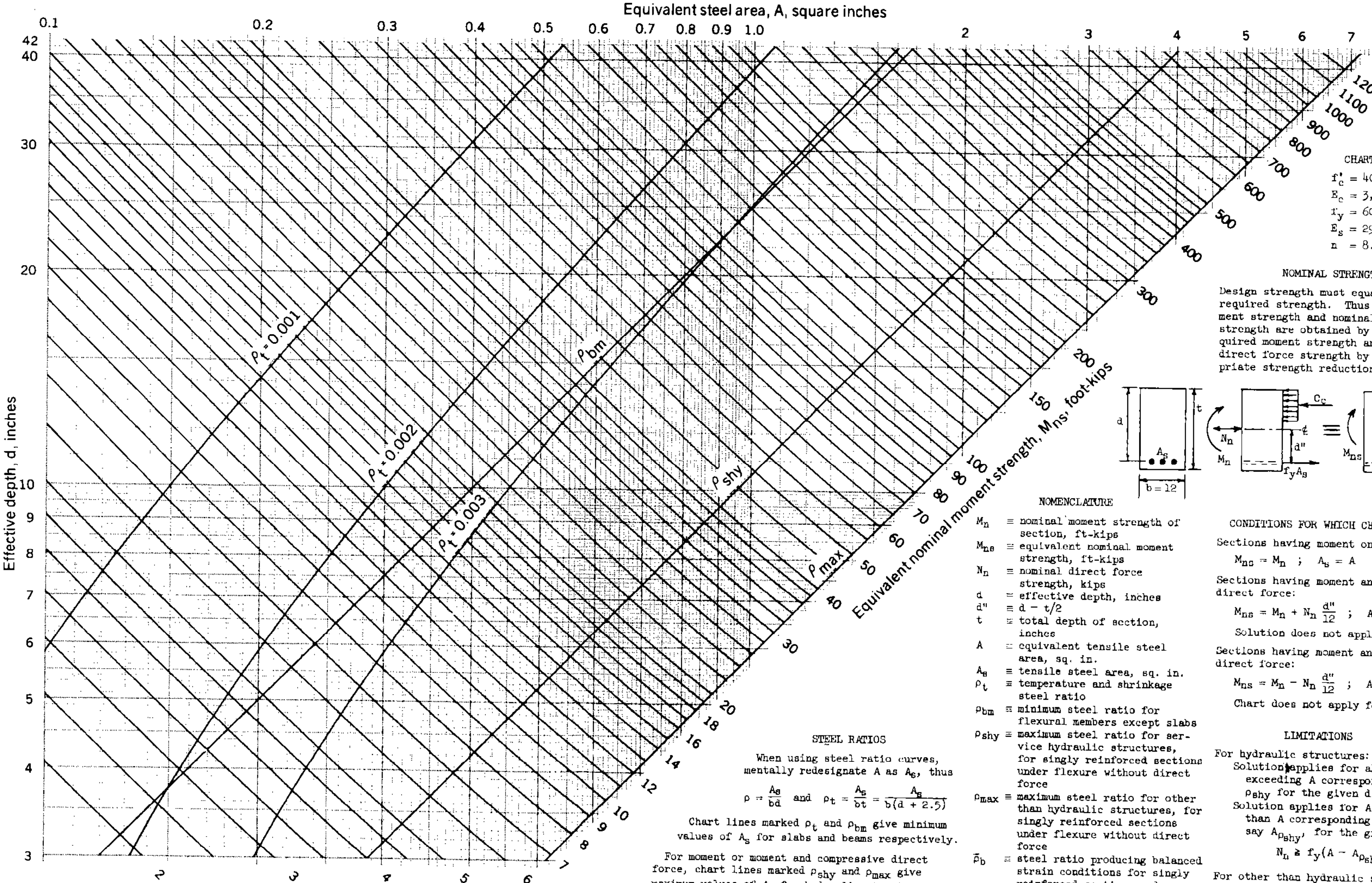


CHART CONSTANTS

$f'_c = 4000$ psi
$E_c = 3,640,000$ psi
$f_y = 60$ ksi
$E_s = 29,000,000$ psi
$n = 8.0$

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.

NOMENCLATURE

M_n ≡ nominal moment strength of section, ft-kips
M_{ns} ≡ equivalent nominal moment strength, ft-kips
N_n ≡ nominal direct force strength, kips
d ≡ effective depth, inches
d" ≡ d − t/2
t ≡ total depth of section, inches
A ≡ equivalent tensile steel area, sq. in.
A_s ≡ tensile steel area, sq. in.
ρ_t ≡ temperature and shrinkage steel ratio
ρ_{bm} ≡ minimum steel ratio for flexural members except slabs
ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
$\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
ϕ ≡ strength reduction factor

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$M_{ns} = M_n$; $A_s = A$

Sections having moment and compressive direct force:

$M_{ns} = M_n + N_n \frac{d''}{12}$; $A_s = A - \frac{N_n}{f_y}$

Solution does not apply if $A_s < 0$.

Sections having moment and tensile direct force:

$M_{ns} = M_n - N_n \frac{d''}{12}$; $A_s = A + \frac{N_n}{f_y}$

Chart does not apply for $M_{ns} < 0$.

LIMITATIONS

For hydraulic structures:
Solution applies for all A not exceeding A corresponding to ρ_{shy} for the given d.
Solution applies for A greater than A corresponding to ρ_{shy}, say $A_{\rho_{shy}}$, for the given d if

$N_n \geq f_y(A - A_{\rho_{shy}})$

For other than hydraulic structures:
Solution applies for all A not exceeding A corresponding to ρ_{max} for the given d.

STEEL RATIOS

When using steel ratio curves, mentally redesignate A as A_s, thus

$$\rho = \frac{A_s}{bd} \quad \text{and} \quad \rho_t = \frac{A_s}{bt} = \frac{A_s}{b(d + 2.5)}$$

Chart lines marked ρ_t and ρ_{bm} give minimum values of A_s for slabs and beams respectively.

For moment or moment and compressive direct force, chart lines marked ρ_{shy} and ρ_{max} give maximum values of A_s for hydraulic structures and other structures respectively.

REFERENCE

U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

STANDARD DWG. NO.
ES-222
SHEET 3 OF 5
DATE 12-78

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
SINGLY REINFORCED SECTIONS, 12 INCH WIDTH

f_y = 60 ksi
f'_c = 5000 psi

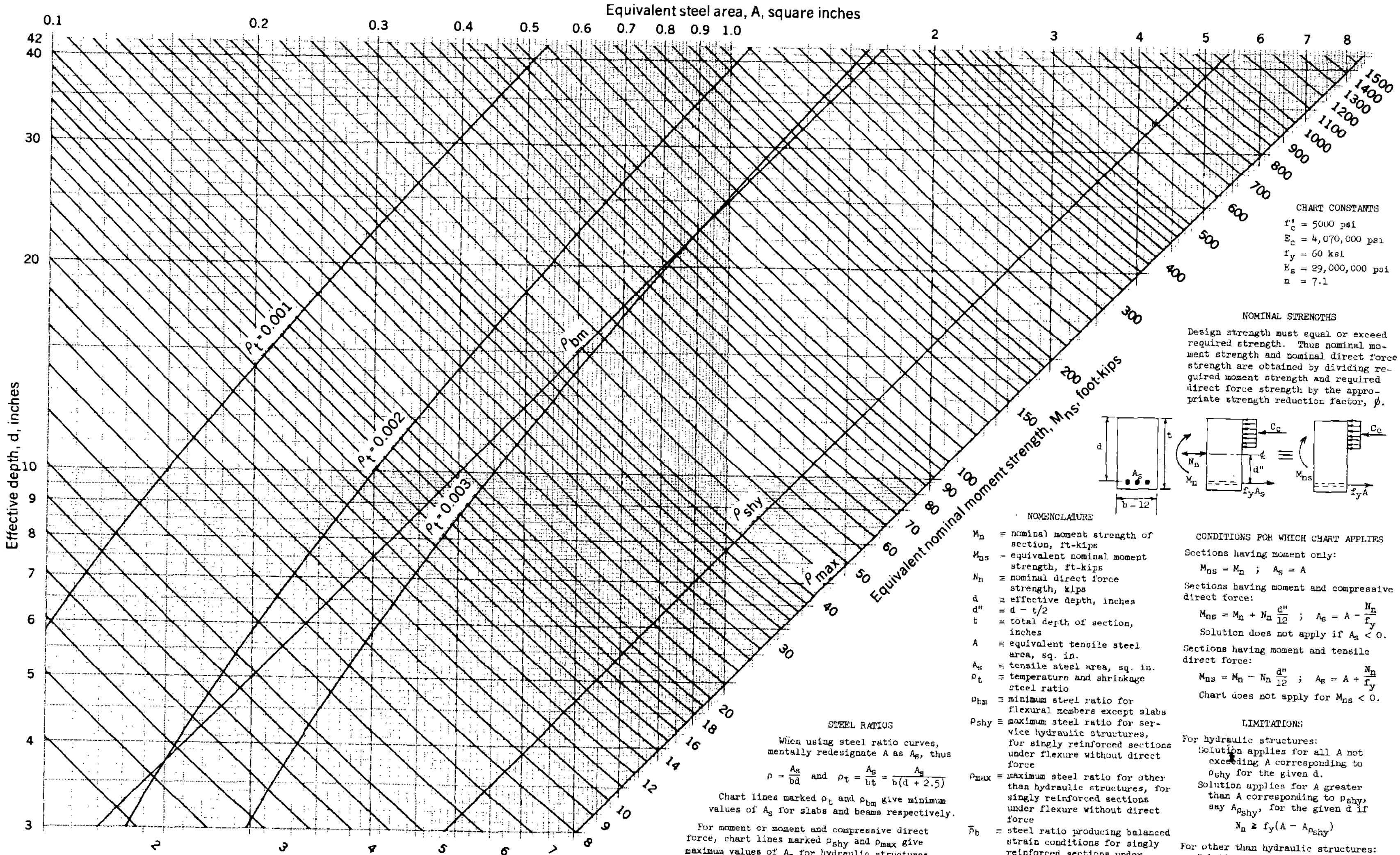


CHART CONSTANTS

f'_c = 5000 psi
E_c = 4,070,000 psi
f_y = 60 ksi
E_s = 29,000,000 psi
n = 7.1

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.

NOMENCLATURE

- M_n = nominal moment strength of section, ft-kips
- M_{ns} = equivalent nominal moment strength, ft-kips
- N_n = nominal direct force strength, kips
- d = effective depth, inches
- d" = d − t/2
- t = total depth of section, inches
- A = equivalent tensile steel area, sq. in.
- A_s = tensile steel area, sq. in.
- ρ_t = temperature and shrinkage steel ratio
- ρ_{bm} = minimum steel ratio for flexural members except slabs
- ρ_{shy} = maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
- ρ_{max} = maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
- $\bar{\rho}_b$ = steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
- ϕ = strength reduction factor

STEEL RATIOS

When using steel ratio curves, mentally redesignate A as A_s, thus

$$\rho = \frac{A_s}{bd} \quad \text{and} \quad \rho_t = \frac{A_s}{bt} = \frac{A_s}{b(d + 2.5)}$$

Chart lines marked ρ_t and ρ_{bm} give minimum values of A_s for slabs and beams respectively.

For moment or moment and compressive direct force, chart lines marked ρ_{shy} and ρ_{max} give maximum values of A_s for hydraulic structures and other structures respectively.

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$$M_{ns} = M_n \quad ; \quad A_s = A$$

Sections having moment and compressive direct force:

$$M_{ns} = M_n + N_n \frac{d''}{12} \quad ; \quad A_s = A - \frac{N_n}{f_y}$$

Solution does not apply if $A_s < 0$.

Sections having moment and tensile direct force:

$$M_{ns} = M_n - N_n \frac{d''}{12} \quad ; \quad A_s = A + \frac{N_n}{f_y}$$

Chart does not apply for $M_{ns} < 0$.

LIMITATIONS

For hydraulic structures:
Solution applies for all A not exceeding A corresponding to ρ_{shy} for the given d.
Solution applies for A greater than A corresponding to ρ_{shy}, say $A_{\rho_{shy}}$, for the given d if

$$N_n \geq f_y(A - A_{\rho_{shy}})$$

For other than hydraulic structures:
Solution applies for all A not exceeding A corresponding to ρ_{max} for the given d.

REFERENCE

U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

STANDARD DWG. NO.
ES-222
SHEET 4 OF 5
DATE 12-78

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
SINGLY REINFORCED SECTIONS, 12 INCH WIDTH

$f_y = 60$ ksi

$f'_c = 6000$ psi

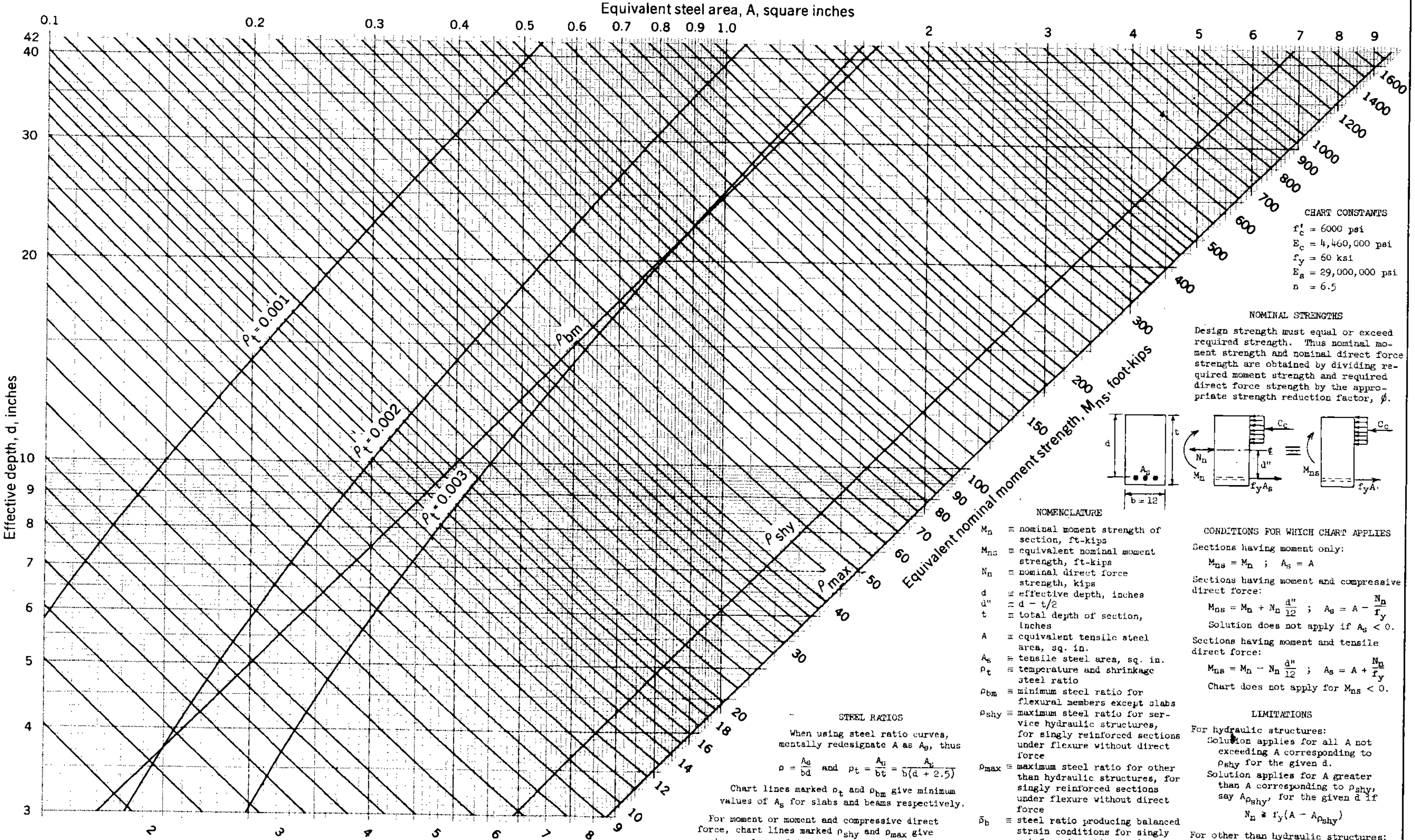


CHART CONSTANTS

$f'_c = 6000$ psi
$E_c = 4,460,000$ psi
$f_y = 60$ ksi
$E_s = 29,000,000$ psi
$n = 6.5$

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.

STEEL RATIOS

When using steel ratio curves, mentally redesignate A as A_s, thus

$$\rho = \frac{A_s}{bd} \quad \text{and} \quad \rho_t = \frac{A_s}{bt} = \frac{A_s}{b(d + 2.5)}$$

Chart lines marked ρ_t and ρ_{bm} give minimum values of A_s for slabs and beams respectively.

For moment or moment and compressive direct force, chart lines marked ρ_{shy} and ρ_{max} give maximum values of A_s for hydraulic structures and other structures respectively.

NOMENCLATURE

- M_n = nominal moment strength of section, ft-kips
- M_{ns} = equivalent nominal moment strength, ft-kips
- N_n = nominal direct force strength, kips
- d = effective depth, inches
- d'' = $d - t/2$
- t = total depth of section, inches
- A = equivalent tensile steel area, sq. in.
- A_s = tensile steel area, sq. in.
- ρ_t = temperature and shrinkage steel ratio
- ρ_{bm} = minimum steel ratio for flexural members except slabs
- ρ_{shy} = maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
- ρ_{max} = maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
- $\bar{\rho}_b$ = steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
- ϕ = strength reduction factor

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$$M_{ns} = M_n \quad ; \quad A_s = A$$

Sections having moment and compressive direct force:

$$M_{ns} = M_n + N_n \frac{d''}{12} \quad ; \quad A_s = A - \frac{N_n}{f_y}$$

Solution does not apply if $A_s < 0$.

Sections having moment and tensile direct force:

$$M_{ns} = M_n - N_n \frac{d''}{12} \quad ; \quad A_s = A + \frac{N_n}{f_y}$$

Chart does not apply for $M_{ns} < 0$.

LIMITATIONS

For hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{shy} for the given d.

Solution applies for A greater than A corresponding to ρ_{shy}, say $A_{\rho_{shy}}$, for the given d if

$$N_n \geq f_y(A - A_{\rho_{shy}})$$

For other than hydraulic structures:

Solution applies for all A not exceeding A corresponding to ρ_{max} for the given d.

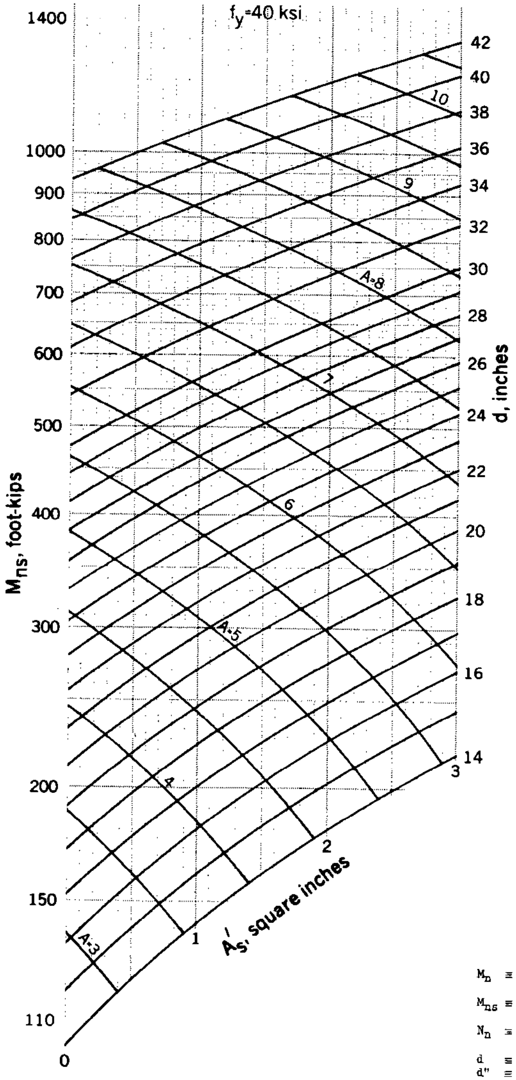
REFERENCE

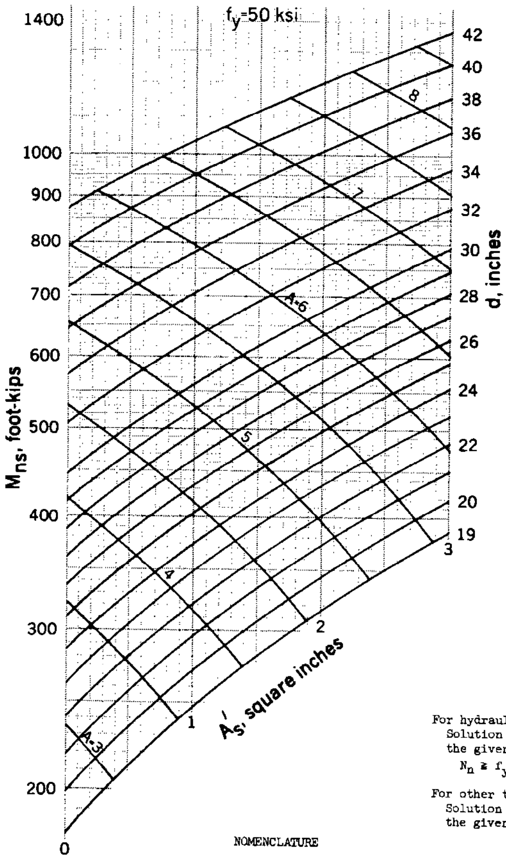
U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

STANDARD DWG. NO.
ES-222
SHEET 5 OF 5
DATE 12-78

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
DOUBLY REINFORCED SECTIONS, 12 INCH WIDTH

$f'_c = 2500$ psi





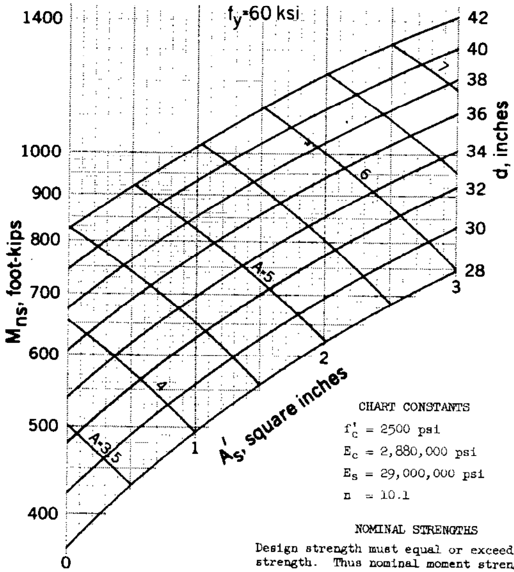
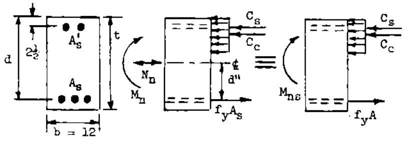


CHART CONSTANTS

f'_c = 2500 psi
E_c = 2,880,000 psi
E_s = 29,000,000 psi
n = 10.1

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.



LIMITATIONS

For hydraulic structures:
Solution applies for all A for the given d if

$N_n \geq f_y(A_{\rho_{max}} - A_{\rho_{shy}})$

For other than hydraulic structures:
Solution applies for all A for the given d.

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$M_{ns} = M_n$; $A_s = A$

Sections having moment and compressive direct force:

$M_{ns} = M_n + N_n \frac{d''}{12}$; $A_s = A - \frac{N_n}{f_y}$

Solution does not apply if $A_s < 0$.

Sections having moment and tensile direct force:

$M_{ns} = M_n - N_n \frac{d''}{12}$; $A_s = A + \frac{N_n}{f_y}$

Chart does not apply for $M_{ns} < 0$.

Chart is drawn for a constant steel area. difference of

$A - A'_s = \frac{1}{2} \bar{\rho}_b bd$

NOMENCLATURE

M_n ≡ nominal moment strength of section, ft-kips
M_{ns} ≡ equivalent nominal moment strength, ft-kips
N_n ≡ nominal direct force strength, kips
d ≡ effective depth, inches
d" ≡ d − t/2
t ≡ total depth of section, inches
A ≡ equivalent tensile steel area, sq. in.
A_s ≡ tensile steel area, sq. in.
A'_s ≡ compressive steel area, sq. in.
$A_{\rho_{shy}}$ ≡ A corresponding to ρ_{shy} for the given d (found on chart for singly reinforced sections)
$A_{\rho_{max}}$ ≡ A corresponding to ρ_{max} for the given d (found on chart for singly reinforced sections)
ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
$\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
ϕ ≡ strength reduction factor

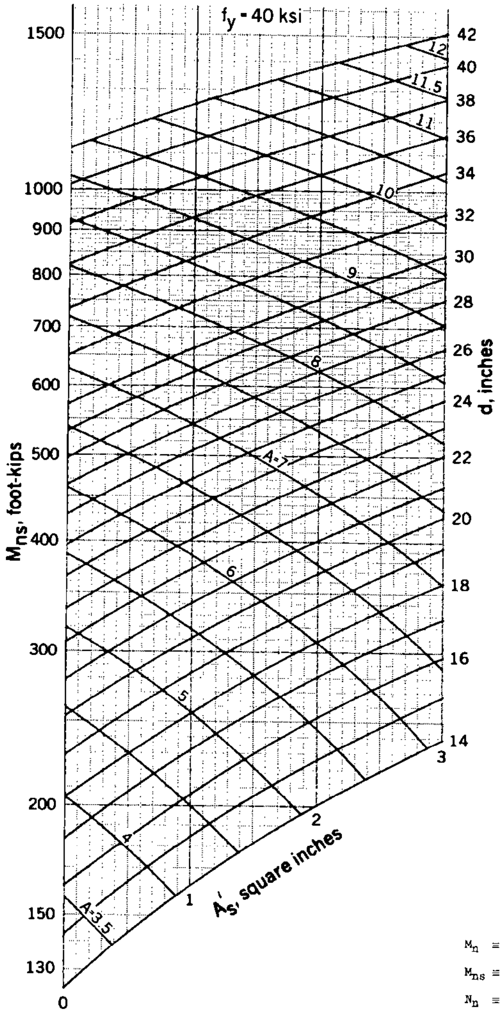
REFERENCE

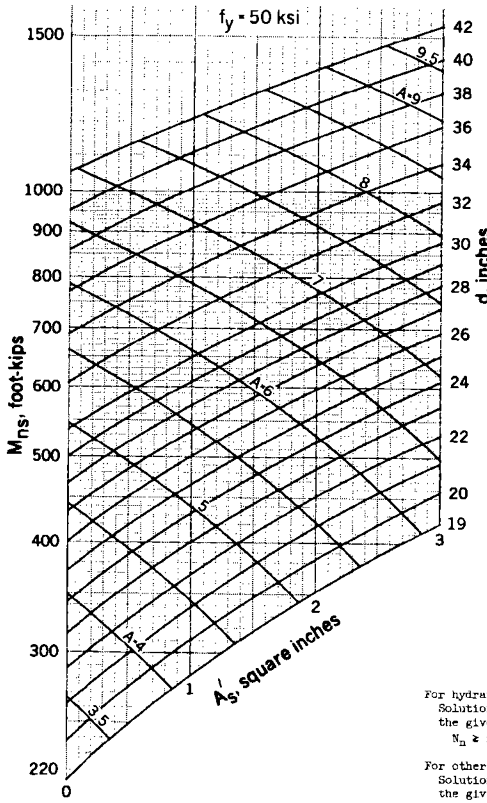
U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION-DESIGN UNIT

STANDARD DWG. NO.
ES-223
SHEET 1 OF 5
DATE 2-79

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
DOUBLY REINFORCED SECTIONS, 12 INCH WIDTH

$f'_c = 3000$ psi





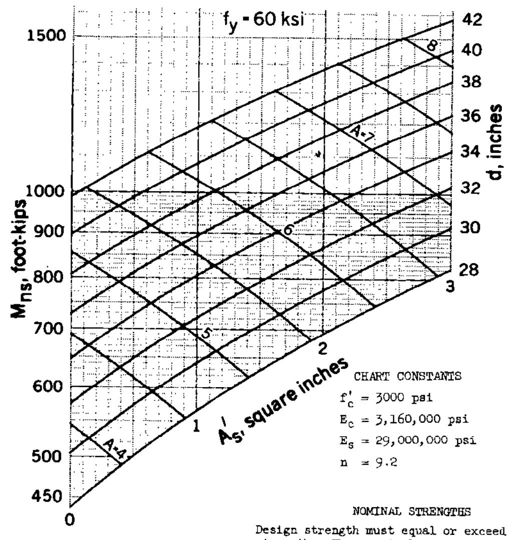


CHART CONSTANTS

$f'_c = 3000$ psi

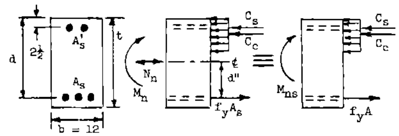
$E_c = 3,160,000$ psi

$E_s = 29,000,000$ psi

$n = 9.2$

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.



LIMITATIONS

For hydraulic structures:
Solution applies for all A for the given d if

$N_n \geq f_y(A_{\rho_{max}} - A_{\rho_{shy}})$

For other than hydraulic structures:
Solution applies for all A for the given d.

NOMENCLATURE

M_n ≡ nominal moment strength of section, ft-kips

M_{ns} ≡ equivalent nominal moment strength, ft-kips

N_n ≡ nominal direct force strength, kips

d ≡ effective depth, inches

d'' ≡ $d - t/2$

t ≡ total depth of section, inches

A ≡ equivalent tensile steel area, sq. in.

A_s ≡ tensile steel area, sq. in.

A'_s ≡ compressive steel area, sq. in.

$A_{\rho_{shy}}$ ≡ A corresponding to ρ_{shy} for the given d (found on chart for singly reinforced sections)

$A_{\rho_{max}}$ ≡ A corresponding to ρ_{max} for the given d (found on chart for singly reinforced sections)

ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force

ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force

$\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force

ϕ ≡ strength reduction factor

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$M_{ns} = M_n$; $A_s = A$

Sections having moment and compressive direct force:

$M_{ns} = M_n + N_n \frac{d''}{12}$; $A_s = A - \frac{N_n}{f_y}$

Solution does not apply if $A_s < 0$.

Sections having moment and tensile direct force:

$M_{ns} = M_n - N_n \frac{d''}{12}$; $A_s = A + \frac{N_n}{f_y}$

Chart does not apply for $M_{ns} < 0$.

Chart is drawn for a constant steel area. difference of

$A - A'_s = \frac{1}{2} \bar{\rho}_b bd$

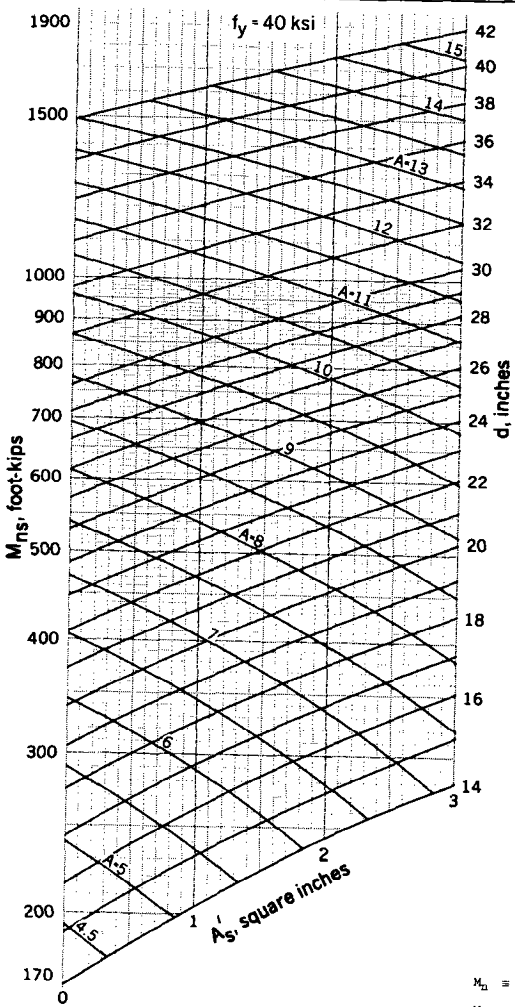
REFERENCE

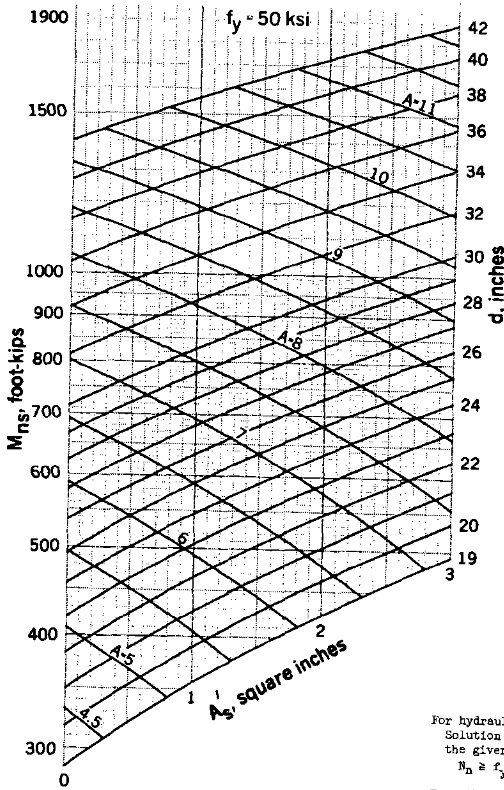
U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION-DESIGN UNIT

STANDARD DWG. NO.
ES-223
SHEET 2 OF 5
DATE 2-79

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
DOUBLY REINFORCED SECTIONS, 12 INCH WIDTH

$f'_c = 4000$ psi





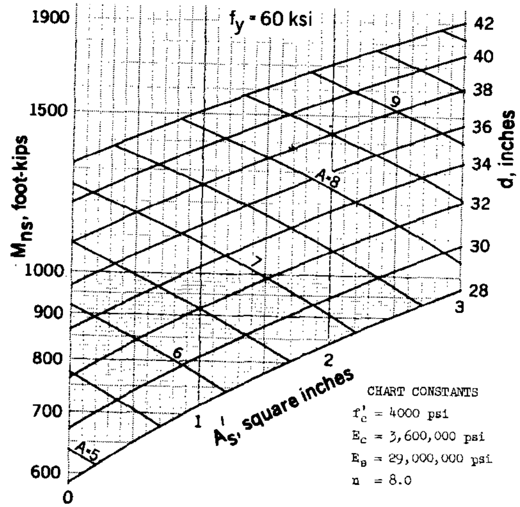


CHART CONSTANTS

$f'_c = 4000$ psi

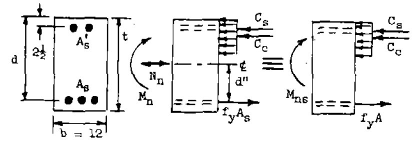
$E_c = 3{,}600{,}000$ psi

$E_s = 29{,}000{,}000$ psi

$n = 8.0$

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.



LIMITATIONS

For hydraulic structures:
Solution applies for all A for the given d if

$N_n \geq f_y(A_{\rho_{max}} - A_{\rho_{shy}})$

For other than hydraulic structures:
Solution applies for all A for the given d.

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$M_{ns} = M_n$; $A_s = A$

Sections having moment and compressive direct force:

$M_{ns} = M_n + N_n \frac{d''}{12}$; $A_s = A - \frac{N_n}{f_y}$

Solution does not apply if $A_s < 0$.

Sections having moment and tensile direct force:

$M_{ns} = M_n - N_n \frac{d''}{12}$; $A_s = A + \frac{N_n}{f_y}$

Chart does not apply for $M_{ns} < 0$.

Chart is drawn for a constant steel area difference of

$A - A'_s = \frac{1}{2} \bar{\rho}_b bd$

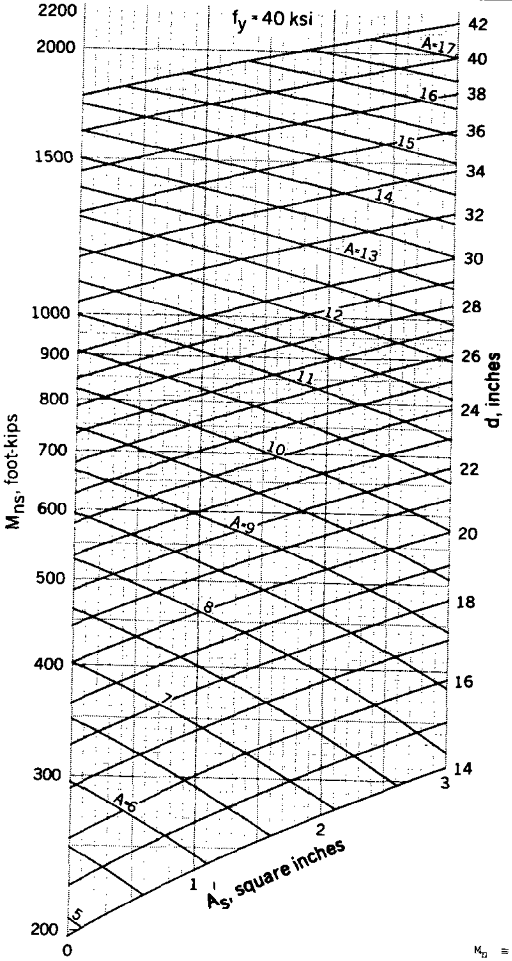
NOMENCLATURE

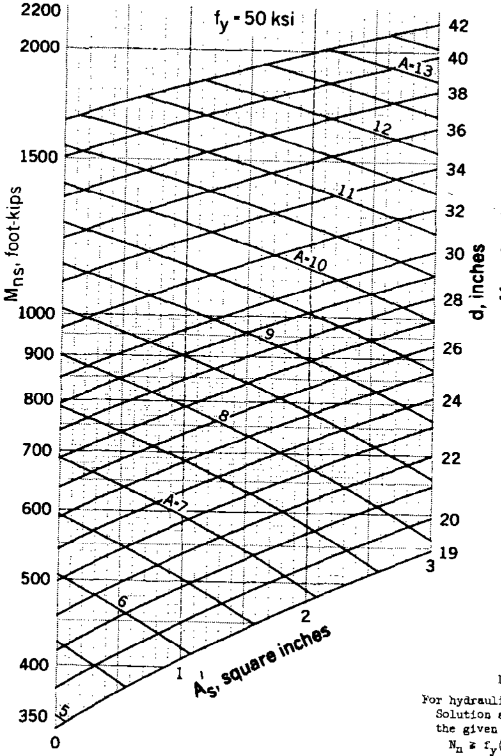
- M_n ≡ nominal moment strength of section, ft-kips
- M_{ns} ≡ equivalent nominal moment strength, ft-kips
- N_n ≡ nominal direct force strength, kips
- d ≡ effective depth, inches
- d'' ≡ $d - t/2$
- t ≡ total depth of section, inches
- A ≡ equivalent tensile steel area, sq. in.
- A_s ≡ tensile steel area, sq. in.
- A'_s ≡ compressive steel area, sq. in.
- $A_{\rho_{shy}}$ ≡ A corresponding to ρ_{shy} for the given d (found on chart for singly reinforced sections)
- $A_{\rho_{max}}$ ≡ A corresponding to ρ_{max} for the given d (found on chart for singly reinforced sections)
- ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
- ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
- $\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
- ϕ ≡ strength reduction factor

REFERENCE

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
DOUBLY REINFORCED SECTIONS, 12 INCH WIDTH

$f'_c = 5000$ psi





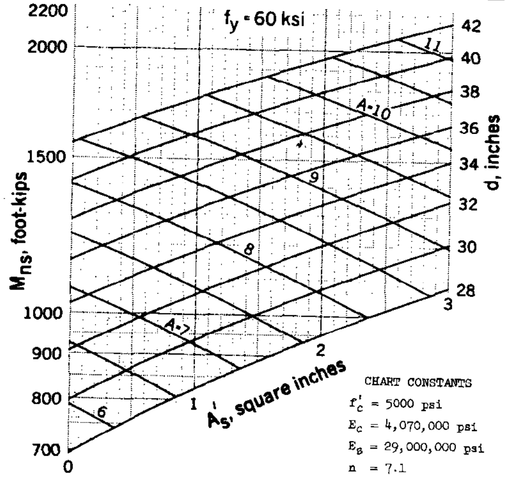


CHART CONSTANTS

f'_c = 5000 psi

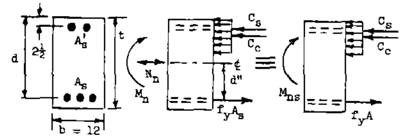
E_c = 4,070,000 psi

E_s = 29,000,000 psi

n = 7.1

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.



LIMITATIONS

For hydraulic structures:
Solution applies for all A for the given d if
$N_n \geq f_y(A_{\rho max} - A_{\rho shy})$

For other than hydraulic structures:
Solution applies for all A for the given d.

NOMENCLATURE

M_n ≡ nominal moment strength of section, ft-kips
M_{ns} ≡ equivalent nominal moment strength, ft-kips
N_n ≡ nominal direct force strength, kips
d ≡ effective depth, inches
d" ≡ d − t/2
t ≡ total depth of section, inches
A ≡ equivalent tensile steel area, sq. in.
A_s ≡ tensile steel area, sq. in.
A'_s ≡ compressive steel area, sq. in.
$A_{\rho shy}$ ≡ A corresponding to ρ_{shy} for the given d (found on chart for singly reinforced sections)
$A_{\rho max}$ ≡ A corresponding to ρ_{max} for the given d (found on chart for singly reinforced sections)
ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
$\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
ϕ ≡ strength reduction factor

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$M_{ns} = M_n$; $A_s = A$

Sections having moment and compressive direct force:

$M_{ns} = M_n + N_n \frac{d''}{12}$; $A_s = A - \frac{N_n}{f_y}$

Solution does not apply if $A_s < 0$.

Sections having moment and tensile direct force:

$M_{ns} = M_n - N_n \frac{d''}{12}$; $A_s = A + \frac{N_n}{f_y}$

Chart does not apply for $M_{ns} < 0$.

Chart is drawn for a constant steel area difference of

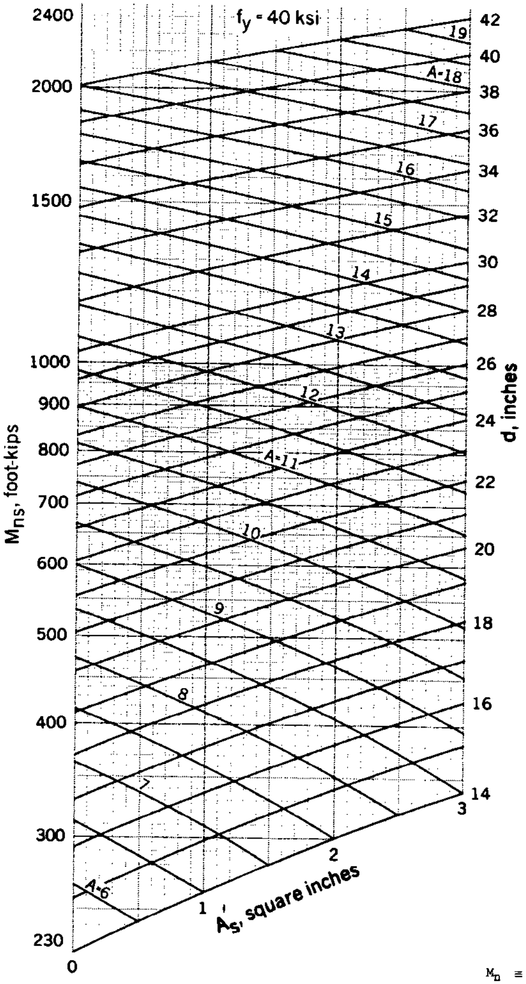
$A - A'_s = \frac{1}{2} \bar{\rho}_b bd$

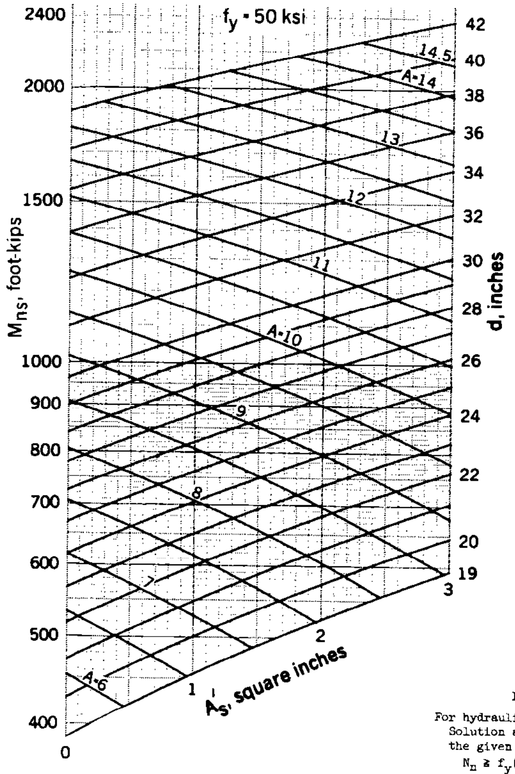
REFERENCE

U.S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION-DESIGN UNIT	STANDARD DWG. NO. ES-223 SHEET 4 OF 5 DATE 2-79

STRUCTURAL DESIGN: REINFORCED CONCRETE DESIGN, STRENGTH DESIGN
BENDING ONLY OR BENDING COMBINED WITH DIRECT FORCE
DOUBLY REINFORCED SECTIONS, 12 INCH WIDTH

$f'_c = 6000$ psi





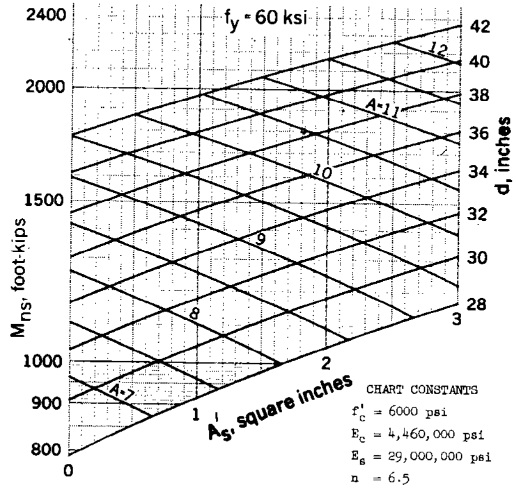
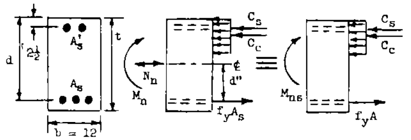


CHART CONSTANTS

f'_c = 6000 psi
E_c = 4,460,000 psi
E_s = 29,000,000 psi
n = 6.5

NOMINAL STRENGTHS

Design strength must equal or exceed required strength. Thus nominal moment strength and nominal direct force strength are obtained by dividing required moment strength and required direct force strength by the appropriate strength reduction factor, ϕ.



LIMITATIONS

For hydraulic structures:
Solution applies for all A for the given d if

$$N_n \geq f_y(A_{\rho_{max}} - A_{\rho_{shy}})$$

For other than hydraulic structures:
Solution applies for all A for the given d.

NOMENCLATURE

M_n ≡ nominal moment strength of section, ft-kips
M_{ns} ≡ equivalent nominal moment strength, ft-kips
N_n ≡ nominal direct force strength, kips
d ≡ effective depth, inches
d" ≡ d − t/2
t ≡ total depth of section, inches
A ≡ equivalent tensile steel area, sq. in.
A_s ≡ tensile steel area, sq. in.
A'_s ≡ compressive steel area, sq. in.
$A_{\rho_{shy}}$ ≡ A corresponding to ρ_{shy} for the given d (found on chart for singly reinforced sections)
$A_{\rho_{max}}$ ≡ A corresponding to ρ_{max} for the given d (found on chart for singly reinforced sections)
ρ_{shy} ≡ maximum steel ratio for service hydraulic structures, for singly reinforced sections under flexure without direct force
ρ_{max} ≡ maximum steel ratio for other than hydraulic structures, for singly reinforced sections under flexure without direct force
$\bar{\rho}_b$ ≡ steel ratio producing balanced strain conditions for singly reinforced sections under flexure without direct force
ϕ ≡ strength reduction factor

CONDITIONS FOR WHICH CHART APPLIES

Sections having moment only:

$$M_{ns} = M_n \; ; \quad A_s = A$$

Sections having moment and compressive direct force:

$$M_{ns} = M_n + N_n \frac{d''}{12} \; ; \quad A_s = A - \frac{N_n}{f_y}$$

Solution does not apply if $A_s < 0$.

Sections having moment and tensile direct force:

$$M_{ns} = M_n - N_n \frac{d''}{12} \; ; \quad A_s = A + \frac{N_n}{f_y}$$

Chart does not apply for $M_{ns} < 0$.

Chart is drawn for a constant steel area difference of

$$A - A'_s = \frac{1}{2} \bar{\rho}_b bd$$

REFERENCE

U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION-DESIGN UNIT

STANDARD DWG. NO.
ES-223
SHEET 5 OF 5
DATE 2-79

STRUCTURAL DESIGN: Reinforced Concrete Design
Strength Design
Development of Reinforcement, Basic Development Lengths

Basic Development Length, ℓ_b, inches

Grade of Steel	Description	Class of Concrete	#3	#4	#5	#6	#7	#8	#9	#10	#11
60	Bars in Tension	6000	9.0*	12.0	15.0	18.0	21.0	24.5	31.0	39.3	48.3
		5000	9.0*	12.0	15.0	18.0	21.0	26.8	33.9	43.1	52.9
		4000	9.0*	12.0	15.0	18.0	22.8	30.0	37.9	48.2	59.2
		3000	9.0*	12.0	15.0	19.3	26.3	34.6	43.8	55.6	68.4
		2500	9.0*	12.0	15.0	21.1	28.8	37.9	48.0	61.0	74.9
	Bars in Compression	6000	6.8+	9.0	11.3	13.5	15.8	18.0	20.3	22.5	24.8
		5000	6.8+	9.0	11.3	13.5	15.8	18.0	20.3	22.5	24.8
		4000	7.1+	9.5	11.9	14.2	16.6	19.0	21.3	23.7	26.1
		3000	8.2	11.0	13.7	16.4	19.2	21.9	24.6	27.4	30.1
		2500	9.0	12.0	15.0	18.0	21.0	24.0	27.0	30.0	33.0
50	Bars in Tension	6000	7.5*	10.0*	12.5	15.0	17.5	20.4	25.8	32.8	40.3
		5000	7.5*	10.0*	12.5	15.0	17.5	22.3	28.3	35.9	44.1
		4000	7.5*	10.0*	12.5	15.0	19.0	25.0	31.6	40.2	49.3
		3000	7.5*	10.0*	12.5	16.1	21.9	28.8	36.5	46.4	57.0
		2500	7.5*	10.0*	12.5	17.6	24.0	31.6	40.0	50.8	62.4
	Bars in Compression	6000	5.6+	7.5+	9.4	11.3	13.1	15.0	16.9	18.8	20.6
		5000	5.6+	7.5+	9.4	11.3	13.1	15.0	16.9	18.8	20.6
		4000	5.9+	7.9+	9.9	11.9	13.8	15.8	17.8	19.8	21.7
		3000	6.8+	9.1	11.4	13.7	16.0	18.3	20.5	22.8	25.1
		2500	7.5+	10.0	12.5	15.0	17.5	20.0	22.5	25.0	27.5
40	Bars in Tension	6000	6.0*	8.0*	10.0*	12.0	14.0	16.3	20.7	26.2	32.2
		5000	6.0*	8.0*	10.0*	12.0	14.0	17.9	22.6	28.7	35.3
		4000	6.0*	8.0*	10.0*	12.0	15.2	20.0	25.3	32.1	39.5
		3000	6.0*	8.0*	10.0*	12.9	17.5	23.1	29.2	37.1	45.6
		2500	6.0*	8.0*	10.0*	14.1	19.2	25.3	32.0	40.6	49.9
	Bars in Compression	6000	4.5+	6.0+	7.5+	9.0	10.5	12.0	13.5	15.0	16.5
		5000	4.5+	6.0+	7.5+	9.0	10.5	12.0	13.5	15.0	16.5
		4000	4.7+	6.3+	7.9+	9.5	11.1	12.6	14.2	15.8	17.4
		3000	5.5+	7.3+	9.1	11.0	12.8	14.6	16.4	18.3	20.1
		2500	6.0+	8.0	10.0	12.0	14.0	16.0	18.0	20.0	22.0

*Tension development length, ℓ_d, shall not be less than 12 inches except in computations of either lap splice lengths or development of web reinforcement lengths.

+Compression development lengths, ℓ_d, shall not be less than 8 inches.

REFERENCE	U.S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN UNIT	STANDARD DWG. NO. ES - 224 SHEET 1 OF 1 DATE 12-79

STRUCTURAL DESIGN: Reinforced Concrete Design
Strength Design
Development of Reinforcement, Standard Hooks in Tension

Equivalent Embedment Length, ℓ_e, inches

Grade of Steel	Description	Class of Concrete	Bar Size #3	#4	#5	#6	#7	#8	#9	#10	#11
60	Tension Top Bars	6000	6.3	8.4	10.5	10.5	10.5*	11.4	14.4	18.3	22.5
		5000	5.7	7.6	9.5	9.5	9.5*	11.4	14.4	18.3	22.5
		4000	5.1	6.8	8.5	8.5	8.6	11.4	14.4	18.3	22.5
		3000	4.5	5.9	7.4	7.9	8.6	11.4	14.4	18.3	22.5
		2500	4.1	5.4	6.8	7.9	8.6	11.4	14.4	18.3	22.5
	Other Tension Bars	6000	6.3	8.4	10.5	12.5	14.6	17.1	21.6	24.4	26.2
		5000	5.7	7.6	9.5	11.5	13.4	17.1	21.6	24.4	26.2
		4000	5.1	6.8	8.5	10.2	13.0	17.1	21.6	24.4	26.2
		3000	4.5	5.9	7.4	9.5	13.0	17.1	21.6	24.4	26.2
		2500	4.1	5.4	6.8	9.5	13.0	17.1	21.6	24.4	26.2
50	Tension Top Bars	6000	5.2	7.0	8.7	9.4	9.8	11.4	14.4	18.3	22.5
		5000	4.8	6.4	8.0	8.6	8.9	11.4	14.4	18.3	22.5
		4000	4.3	6.1	7.1	7.7	8.6	11.4	14.4	18.3	22.5
		3000	3.7	4.9	6.2	7.1	8.6	11.4	14.4	18.3	22.5
		2500	3.4	4.5	5.6	7.1	8.6	11.4	14.4	18.3	22.5
	Other Tension Bars	6000	5.2	7.0	8.7	10.5	12.2	14.2	18.0	21.3	24.3
		5000	4.8	6.4	8.0	9.5	11.1	14.2	18.0	21.3	24.3
		4000	4.3	5.7	7.1	8.5	10.8	14.2	18.0	21.3	24.3
		3000	3.7	4.9	6.2	7.9	10.8	14.2	18.0	21.3	24.3
		2500	3.4	4.5	5.6	7.9	10.8	14.2	18.0	21.3	24.3
40	All Tension	6000	4.2	5.6	7.0	8.4	9.8	11.4	14.4	18.3	22.5
		5000	3.8	5.1	6.4	7.6	8.9	11.4	14.4	18.3	22.5
		4000	3.4	4.6	5.7	6.8	8.6	11.4	14.4	18.3	22.5
		3000	3.0	3.9	4.9	6.3	8.6	11.4	14.4	18.3	22.5
		2500	2.7	3.6	4.5	6.3	8.6	11.4	14.4	18.3	22.5

*For consistency, the value modified slightly from computed.

STRUCTURAL DESIGN: Reinforced Concrete Design
Strength Design
Development of Reinforcement, Standard Hooks in Tension

Developed Tensile Stress, $f_h = \xi \sqrt{f'_c}/1000$ ksi

Grade of Steel	Description	Class of Concrete	Bar Size #3	#4	#5	#6	#7	#8	#9	#10	#11
60	Tension Top Bars	6000	41.8	41.8	41.8	34.9	27.9	27.9	27.9	27.9	27.9
		5000	38.2	38.2	38.2	31.8	25.5	25.5	25.5	25.5	25.5
		4000	34.2	34.2	34.2	28.5	22.8	22.8	22.8	22.8	22.8
		3000	29.7	29.7	29.7	24.6	19.7	19.7	19.7	19.7	19.7
		2500	27.0	27.0	27.0	22.5	18.0	18.0	18.0	18.0	18.0
	Other Tension Bars	6000	41.8	41.8	41.8	41.8	41.8	41.8	41.8	37.2	32.5
		5000	38.2	38.2	38.2	38.2	38.2	38.2	38.2	33.9	29.7
		4000	34.2	34.2	34.2	34.2	34.2	34.2	34.2	30.4	26.6
		3000	29.7	29.7	29.7	29.7	29.7	29.7	29.7	26.3	23.0
		2500	27.0	27.0	27.0	27.0	27.0	27.0	27.0	24.0	21.0
50	Tension Top Bars	6000	34.9	34.9	34.9	31.4	27.9	27.9	27.9	27.9	27.9
		5000	31.8	31.8	31.8	28.6	25.5	25.5	25.5	25.5	25.5
		4000	28.5	28.5	28.5	25.6	22.8	22.8	22.8	22.8	22.8
		3000	24.6	24.6	24.6	22.2	19.7	19.7	19.7	19.7	19.7
		2500	22.5	22.5	22.5	20.3	18.0	18.0	18.0	18.0	18.0
	Other Tension Bars	6000	34.9	34.9	34.9	34.9	34.9	34.9	34.9	32.5	30.2
		5000	31.8	31.8	31.8	31.8	31.8	31.8	31.8	29.7	27.6
		4000	28.5	28.5	28.5	28.5	28.5	28.5	28.5	26.6	24.7
		3000	24.6	24.6	24.6	24.6	24.6	24.6	24.6	23.0	21.4
		2500	22.5	22.5	22.5	22.5	22.5	22.5	22.5	21.0	19.5
40	All Tension Bars	6000	27.9	27.9	27.9	27.9	27.9	27.9	27.9	27.9	27.9
		5000	25.5	25.5	25.5	25.5	25.5	25.5	25.5	25.5	25.5
		4000	22.8	22.8	22.8	22.8	22.8	22.8	22.8	22.8	22.8
		3000	19.7	19.7	19.7	19.7	19.7	19.7	19.7	19.7	19.7
		2500	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0	18.0

STRUCTURAL DESIGN: Reinforced Concrete Design
Strength Design
Development of Reinforcement, Standard Hooks in Tension

Stress Coefficient for Tensile Stress, ζ values in $f_h = \zeta \sqrt{f'_c}$

Grade of Steel	Description	Class of Concrete	Bar Size #3	#4	#5	#6	#7	#8	#9	#10	#11
60	Tension Top Bars	6000 5000 4000 3000 2500	540	540	540	450	360	360	360	360	360
	Other Tension Bars	6000 5000 4000 3000 2500	540	540	540	540	540	540	540	480	420
50	Tension Top Bars	6000 5000 4000 3000 2500	450	450	450	405	360	360	360	360	360
	Other Tension Bars	6000 5000 4000 3000 2500	450	450	450	450	450	450	450	420	390
40	All Tension Bars	6000 5000 4000 3000 2500	360	360	360	360	360	360	360	360	360

STRUCTURAL DESIGN: Reinforced Concrete Design
Strength Design
Development of Reinforcement, Web Reinforcement

Pertinent Embedment Lengths for Web Reinforcement, inches

Grade of Steel	Class of Concrete	Description	Bar Size #3	#4	#5	#6	#7	Description	Bar Size #3	#4	#5	#6	#7
60	6000	$1.7\ \ell_d$	16	21	26	31	36	$\frac{1}{2}\ \ell_d$	5	6	8	9	11
	5000		16	21	26	31	36		5	6	8	9	11
	4000		16	21	26	31	36		5	6	8	9	11
	3000		16	21	26	33	45		5	6	8	10	14
	2500		16	21	26	36	49		5	6	8	11	15
	6000	larger of ℓ_d, 24 d_b, or 12	12	12	15	18	21	$\frac{1}{3}\ \ell_d$	3	4	5	–	–
	5000		12	12	15	18	21		3	4	5	–	–
	4000		12	12	15	18	21		3	4	5	–	–
	3000		12	12	15	20	27		3	4	5	–	–
	2500		12	12	15	22	29		3	4	5	–	–
50	6000	$1.7\ \ell_d$	13	17	22	26	30	$\frac{1}{2}\ \ell_d$	4	5	7	8	9
	5000		13	17	22	26	30		4	5	7	8	9
	4000		13	17	22	26	33		4	5	7	8	10
	3000		13	17	22	28	38		4	5	7	9	11
	2500		13	17	22	30	41		4	5	7	9	12
	6000	larger of ℓ_d, 24 d_b, or 12	12	12	15	18	21	$\frac{1}{3}\ \ell_d$	3	4	5	–	–
	5000		12	12	15	18	21		3	4	5	–	–
	4000		12	12	15	18	21		3	4	5	–	–
	3000		12	12	15	18	22		3	4	5	–	–
	2500		12	12	15	18	24		3	4	5	–	–
40	6000	$1.7\ \ell_d$	12	14	17	21	24	$\frac{1}{2}\ \ell_d$	3	4	5	6	7
	5000		12	14	17	21	24		3	4	5	6	7
	4000		12	14	17	21	26		3	4	5	6	8
	3000		12	14	17	22	30		3	4	5	7	9
	2500		12	14	17	24	33		3	4	5	8	10
	6000	larger of ℓ_d, 24 d_b, or 12	12	12	15	18	21	$\frac{1}{3}\ \ell_d$	2	3	4	–	–
	5000		12	12	15	18	21		2	3	4	–	–
	4000		12	12	15	18	21		2	3	4	–	–
	3000		12	12	15	18	21		2	3	4	–	–
	2500		12	12	15	18	21		2	3	4	–	–

Note: ℓ_d is taken as ℓ_b from ES-224

STRUCTURAL DESIGN: Reinforced Concrete Design
Strength Design
Lap Splices in Reinforcement

Lapped Splice Lengths, inches												
Grade of Steel	Description	Class of Concrete	Class of Splice	Bar Size								
				#3	#4	#5	#6	#7	#8	#9	#10	#11
60	Tension Top Bars	6000	A	13	17	21	26	30	35	44	56	68
			B	17	22	28	33	39	46	57	72	88
			C	22	29	36	43	50	59	74	94	115
		5000	A	13	17	21	26	30	38	48	61	75
			B	17	22	28	33	39	49	62	79	97
			C	22	29	36	43	50	64	81	103	126
		4000	A	13	17	21	26	32	42	54	68	83
			B	17	22	28	33	42	55	69	88	108
			C	22	29	36	43	55	72	91	115	141
		3000	A	13	17	21	28	37	49	62	78	96
			B	17	22	28	36	48	63	80	102	125
			C	22	29	36	46	63	83	104	133	163
		2500	A	13	17	21	30	41	54	68	86	105
			B	17	22	28	39	53	69	88	111	137
			C	22	29	36	51	69	91	115	146	179
	All Other Tension Bars	6000	A	12	15	19	23	27	30	34	40	49
			B	16	16	20	24	28	32	41	52	63
			C	21	21	26	31	36	42	53	67	83
		5000	A	12	15	19	23	27	30	34	44	53
			B	16	16	20	24	28	35	45	57	69
			C	21	21	26	31	36	46	58	74	90
		4000	A	12	15	19	23	27	30	38	49	60
			B	16	16	20	24	30	39	50	63	77
			C	21	21	26	31	39	51	65	82	101
		3000	A	12	15	19	23	27	35	44	56	69
			B	16	16	20	26	35	45	57	73	89
			C	21	21	26	33	45	59	75	95	117
		2500	A	12	15	19	23	29	38	48	61	75
			B	16	16	20	28	38	50	63	80	98
			C	21	21	26	36	49	65	82	104	128
	All Compression Bars	6000		12	15	19	23	27	30	34	38	42
		5000		12	15	19	23	27	30	34	38	42
		4000		12	15	19	23	27	30	34	38	42
		3000		12	15	19	23	27	30	34	38	42
		2500		16	20	26	31	36	40	46	51	56

Tension bars spaced laterally not less than 6 inches on center, and bars with at least 3 inches clear from face of member to first bar may use 0.8 lap lengths shown but not less than 12 inches.

STRUCTURAL DESIGN: Reinforced Concrete Design
Strength Design
Lap Splices in Reinforcement

Lapped Splice Lengths, inches

Grade of Steel	Description	Class of Concrete	Class of Splice	#3	#4	#5	#6	#7	#8	#9	#10	#11
50	Tension Top Bars	6000	A	12	15	19	23	27	30	37	46	57
			B	16	19	23	28	32	38	47	60	74
			C	21	24	30	36	42	49	62	79	96
		5000	A	12	15	19	23	27	32	40	51	62
			B	16	19	23	28	32	41	52	66	81
			C	21	24	30	36	42	54	68	86	105
		4000	A	12	15	19	23	27	35	45	57	70
			B	16	19	23	28	35	46	58	73	90
			C	21	24	30	36	46	60	76	96	118
		3000	A	12	15	19	23	31	41	52	65	80
			B	16	19	23	30	40	53	67	85	104
			C	21	24	30	39	53	69	87	111	136
		2500	A	12	15	19	25	35	45	56	72	88
			B	16	19	23	33	45	58	73	93	114
			C	21	24	30	42	59	76	96	121	149
	All Other Tension Bars	6000	A	12	15	19	23	27	30	34	38	42
			B	16	16	19	23	27	30	34	43	53
			C	21	21	22	26	30	35	44	56	69
		5000	A	12	15	19	23	27	30	34	38	45
			B	16	16	19	23	27	30	37	47	58
			C	21	21	22	26	30	38	49	62	75
		4000	A	12	15	19	23	27	30	34	41	50
			B	16	16	19	23	27	33	42	53	65
			C	21	21	22	26	33	43	54	69	84
		3000	A	12	15	19	23	27	30	37	47	57
			B	16	16	19	23	29	38	48	61	75
			C	21	21	22	28	38	49	63	79	97
		2500	A	12	15	19	23	27	32	40	51	63
			B	16	16	19	23	32	42	52	67	82
			C	21	21	22	30	42	54	68	87	107
	All Compression Bars	6000		12	13	16	19	22	25	29	32	35
		5000		12	13	16	19	22	25	29	32	35
		4000		12	13	16	19	22	25	29	32	35
		3000		12	13	16	19	22	25	29	32	35
		2500		16	18	22	26	30	34	39	43	47

Tension bars spaced laterally not less than 6 inches on center, and bars with at least 3 inches clear from face of member to first bar may use 0.8 lap lengths shown but not less than 12 inches.

STRUCTURAL DESIGN: Reinforced Concrete Design
Strength Design
Lap Splices in Reinforcement

Lapped Splice Lengths, inches

Grade of Steel	Description	Class of Concrete	Class of Splice	Bar Size #3	#4	#5	#6	#7	#8	#9	#10	#11
40	Tension Top Bars	6000	A	12	15	19	23	27	30	34	38	46
			B	16	16	19	23	27	30	38	48	59
			C	21	21	24	29	34	39	50	63	77
		5000	A	12	15	19	23	27	30	34	41	50
			B	16	16	19	23	27	33	42	53	65
			C	21	21	24	29	34	43	54	69	85
		4000	A	12	15	19	23	27	30	36	45	56
			B	16	16	19	23	28	37	47	59	72
			C	21	21	24	29	37	48	61	77	95
		3000	A	12	15	19	23	27	33	41	52	64
			B	16	16	19	24	32	43	54	68	83
			C	21	21	24	31	42	55	70	89	109
		2500	A	12	15	19	23	27	36	45	57	70
			B	16	16	19	26	35	47	59	74	91
			C	21	21	24	34	46	61	77	97	119
	All Other Tension Bars	6000	A	12	15	19	23	27	30	34	38	42
			B	16	16	19	23	27	30	34	38	42
			C	21	21	21	23	27	30	36	45	55
		5000	A	12	15	19	23	27	30	34	38	42
			B	16	16	19	23	27	30	34	38	46
			C	21	21	21	23	27	31	39	49	61
		4000	A	12	15	19	23	27	30	34	38	42
			B	16	16	19	23	27	30	34	42	52
			C	21	21	21	23	27	34	44	55	68
		3000	A	12	15	19	23	27	30	34	38	46
			B	16	16	19	23	27	31	38	49	60
			C	21	21	21	23	30	40	50	64	78
		2500	A	12	15	19	23	27	30	34	41	50
			B	16	16	19	23	27	33	42	53	65
			C	21	21	21	24	33	44	55	70	85
	All Compression Bars	6000		12	12	15	18	21	24	27	30	33
		5000		12	12	15	18	21	24	27	30	33
		4000		12	12	15	18	21	24	27	30	33
		3000		12	12	15	18	21	24	27	30	33
		2500		16	16	20	24	28	32	36	40	44

Tension bars spaced laterally not less than 6 inches on center, and bars with at least 3 inches clear from face of member to first bar may use 0.8 lap lengths shown but not less than 12 inches.

REFERENCE

U.S. DEPARTMENT OF AGRICULTURE
SOIL CONSERVATION SERVICE
ENGINEERING DIVISION - DESIGN UNIT

STANDARD DWG. NO.
ES - 227

DATE 12-79

STRUCTURAL DESIGN: Reinforced Concrete Design
Strength Design
Control of Flexural Cracking, One-way Slabs

Maximum Bar Spacing, S, inches

Clear Cover = 2 inches

Bar Size	Z = 145, f_s, ksi						Z = 130, f_s, ksi						Bar Size
	15	20	25	30	35	40	15	20	25	30	35	40	
#3	>18	>18	>18	11.8	7.4	5.0	>18	>18	14.7	8.5	5.4	3.6	#3
#4	>18	>18	>18	11.2	7.0	4.7	>18	>18	13.9	8.0	5.1	3.4	#4
#5	>18	>18	>18	10.6	6.6	4.5	>18	>18	13.1	7.6	4.8	3.2	#5
#6	>18	>18	17.3	10.0	6.3	4.2	>18	>18	12.5	7.2	4.5	3.0	#6
#7	>18	>18	16.4	9.5	6.0	4.0	>18	>18	11.8	6.8	4.3	2.9	#7
#8	>18	>18	15.6	9.0	5.7	3.8	>18	>18	11.2	6.5	4.1	2.7	#8
#9	>18	>18	14.9	8.6	5.4	3.6	>18	>18	10.7	6.2	3.9	2.6	#9
#10	>18	>18	14.2	8.2	5.2	3.5	>18	>18	10.2	5.9	3.7	–	#10
#11	>18	>18	13.5	7.8	4.9	3.3	>18	>18	9.7	5.6	3.5	–	#11

Clear Cover = 3 inches

Bar Size	Z = 145, f_s, ksi						Z = 130, f_s, ksi						Bar Size
	15	20	25	30	35	40	15	20	25	30	35	40	
#3	>18	>18	9.6	5.6	3.5	2.3	>18	13.5	6.9	4.0	2.5	1.7	#3
#4	>18	18.0	9.2	5.3	3.4	2.3	>18	13.0	6.7	3.9	2.4	1.6	#4
#5	>18	17.4	8.9	5.1	3.2	2.2	>18	12.5	6.4	3.7	2.3	–	#5
#6	>18	16.7	8.6	5.0	3.1	2.1	>18	12.1	6.2	3.6	2.2	–	#6
#7	>18	16.1	8.3	4.8	3.0	2.0	>18	11.6	5.9	3.4	2.2	–	#7
#8	>18	15.6	8.0	4.6	2.9	–	>18	11.2	5.7	3.3	2.1	–	#8
#9	>18	15.0	7.7	4.4	2.8	–	>18	10.8	5.5	3.2	–	–	#9
#10	>18	14.5	7.4	4.3	2.7	–	>18	10.4	5.4	3.1	–	–	#10
#11	>18	14.0	7.2	4.2	–	–	>18	10.1	5.2	3.0	–	–	#11

REFERENCE	U.S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN UNIT	STANDARD DWG. NO. ES - 228 SHEET 1 OF 1 DATE 12-79

*U.S. GOVERNMENT PRINTING OFFICE: 1990--726-235/20773